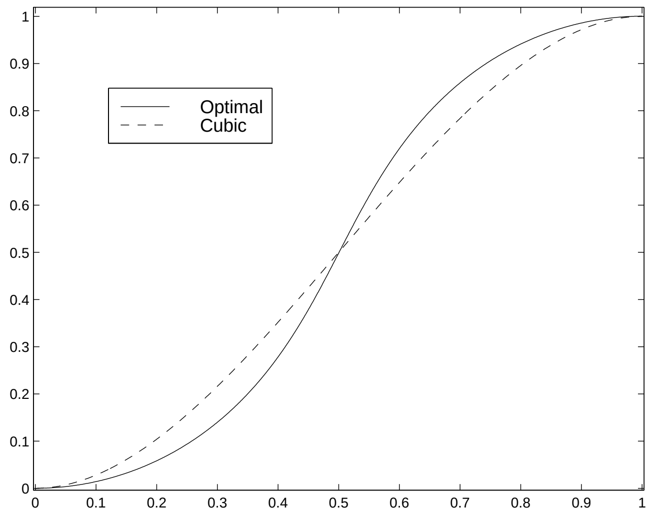
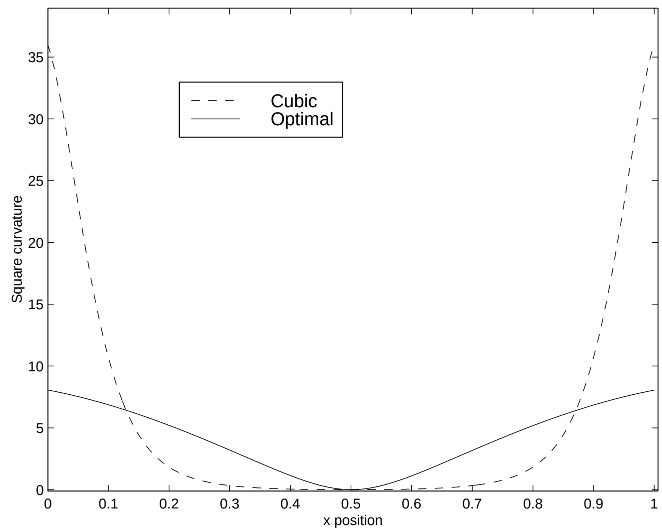


5.19.Optimal and cubic spline paths to the configuration $(1, 1, 0)$, $\beta = 0$.

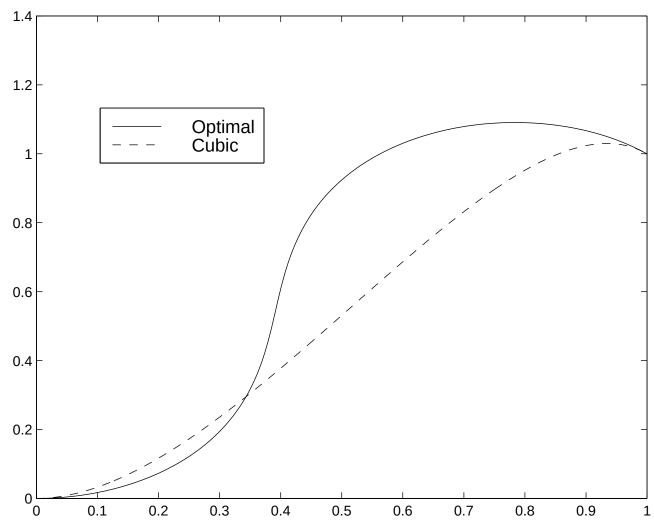
being J_S computed according to

- **STEP** *(i)* : initialize ks sign at the starting configuration $(0, 0, 0)$
- **STEP** *(ii)* : integrate (Euler integration has been employed in the worked examples) equation (5.51) with the sign fixed at step *(i)* from $\theta = 0$ to $\theta = \theta^*$, being θ^* such that $a_1 \cos\theta^* + a_2 \sin\theta^* = 0$, and from θ^* to θ_f with the opposite sign.
- **STEP** *(iii)* : having computed $\theta(t)$ at step *(ii)* such that $\theta(0) = 0$ and $\theta(t_f) = \theta_f$ integrate equations (5.49) and (5.50) to yield $\bar{x}(\mathbf{a}, \theta_f) = \int_{\mathbf{0}}^{t_f} \cos\theta(t)dt$ and $\bar{y}(\mathbf{a}, \theta_f) = \int_{\mathbf{0}}^{t_f} \sin\theta(t)dt$
- **STEP** *(iii)* : $J_S = (x_f - \bar{x}(\mathbf{a}, \theta_f))^2 + (y_f - \bar{y}(\mathbf{a}, \theta_f))^2$.

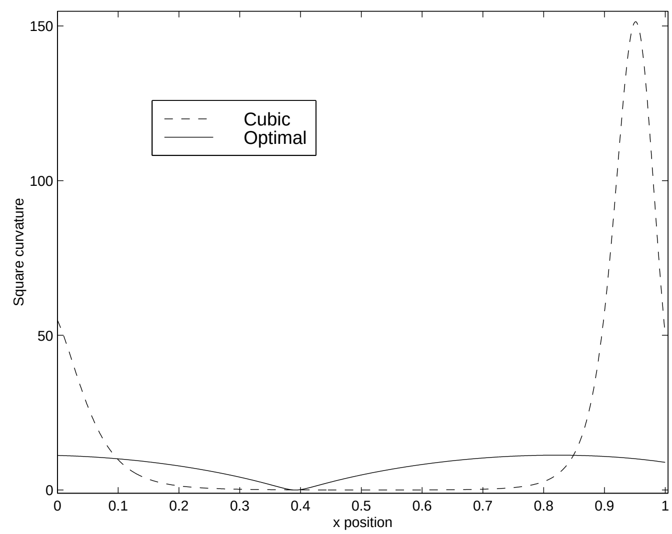
Examples of paths computed according to this algorithm are given in figures (5.19) and (5.21). In the reported examples the minimization of J_S has been performed by the simplex method provided by the Matlab software. Notice that in all the presented examples the square curvature of the optimal paths is dramatically lower then the cubic spline square curvature over the whole path. Cubic splines have been used to compare the optimal solution behaviour as they are the least order polynomials satisfying the given boundary configurations. Although these numerical solution methods are straight forward they are not suitable for on line path planning algorithms thus future work will investigate alternative approaches to the calculation of the constants in equation (5.44).



5.20.Square curvature versus the x position of the cubic spline and optimal paths having $(1,1,0)$ as final configuration; $\beta = 0$.



5.21.Cubic spline and optimal paths for the $(1,1,-\pi/4)$ final configuration with $\beta = 0$.



5.22.Square curvature versus x position for the cubic spline and optimal paths relative to the final configuration $(1, 1, -\pi/4)$.

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