GUIDANCE OF 3D UNDERWATER NON-HOLONOMIC VEHICLE VIA PROJECTION ON HOLONOMIC SOLUTIONS

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ABSTRACT

The feedback control of a nonholonomic 3D floating vehicle is considered: namely the control objective is to drive a vehicle moving in 3D space to a given point and heading along a given line having as control inputs a 1D linear velocity (surge velocity) and a 2D angular one allowing the vehicle to rotate around any axis normal to the surge one. This kind of kinematic describes a large class of floating systems including underwater or space vehicles. In this paper it is shown that a suitable choice of state variables, i.e. a polar-like set of coordinates, allows to easily determine a nonlinear time-invariant closed loop law that drives the configuration error to zero as long as the vehicles is not initially positioned in the target point. The configuration error is shown to tend asymptotically towards zero via a Lypaunov analysis. The proposed solution builds on previous work regarding the planar unicycle kinematic model and its effectiveness is confirmed by a simulation analysis.

KEYWORDS: Nonholonomic vehicles, position control

INTRODUCTION

The underactuated degrees of freedom of a rigid body moving in 3D space determine nonintegrable, i.e. nonholonomic, constraints. This is the case, by example, of an underwater vehicle having nonactuated sway, heave and roll which is by far the most common case in real applications. With reference to figure (1) consider a rigid body having a body fixed reference $<b>$ such that the linear and angular velocities obey

$$\underline{u} = u\,\underline{i}_b \tag{1}$$

$$\underline{\omega} \cdot \underline{i}_b = 0 \tag{2}$$

As for the planar unicycle case [1], [2], [3] and in spite of Brocketts Theorem it has been shown [4] that also in the 3D case a polar-like description of the system kinematic allows a time-invariant closed loop control law to be found that guarantees asymptotic convergence of the position and orientation error to zero as long as the system is not initially located in the target position. For a more detailed discussion of the controllability of such a kind of underactuated kinematic model refer to [5]. The present paper suggests a more natural approach for the issue of designing a non linear control law for the polar-like description of the underactuated 3D system. The proposed control scheme is obtained as the result of a two stage process: first a velocity vector field is defined such that an ideal system free of any nonholonomic constraint subject to this velocity would exponentially converge to the desired configuration and then a steering law is computed such that the only actuated linear motion axis of the nonholonomic system is asymptotically parallel to the previously defined velocity vector field.

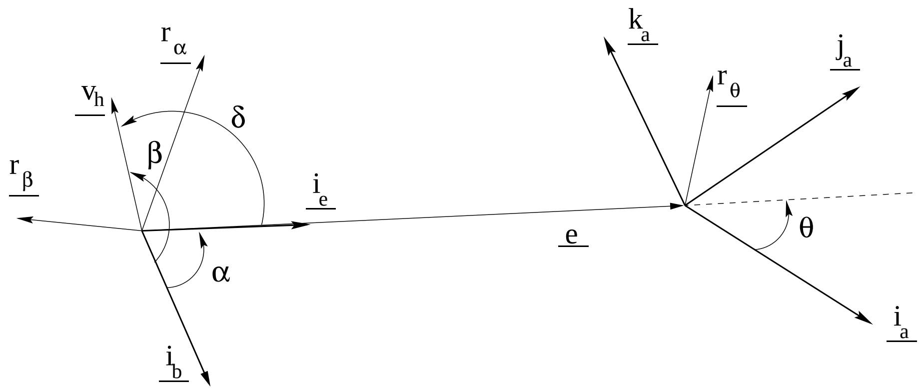

Figure 1: The model

CONTROL LAW DESIGN

With reference to figure (1) consider the body and target fixed frames $<b>=\{\underline{i}_b, \underline{j}_b, \underline{k}_b\}$ and $<a>=\{\underline{i}_a, \underline{j}_a, \underline{k}_a\}$ and the vector $\underline{e}$ pointing from $<b>$ to $<a>$. The angles between $\underline{i}_a$, $\underline{i}_b$ and $\underline{e}$ are θ and α such that when they are different from 0 or π the unit vectors $\underline{r}_\theta$ and $\underline{r}_\alpha$ parallel to $\underline{i}_a \wedge \underline{i}_e$ and $\underline{i}_b \wedge \underline{i}_e$ and normal to the planes $\{\underline{i}_a, \underline{e}\}$ and $\{\underline{i}_b, \underline{e}\}$ are well defined. Calling $\underline{v}_h$ a velocity vector to be yet defined, β is the angle between $\underline{i}_b$ and $\underline{v}_h$ and $\underline{r}_\beta \parallel \underline{i}_b \wedge \underline{v}_h$ the unit vector normal to the plane $\{\underline{i}_b, \underline{v}_h\}$ when $\beta \neq 0, \pi$. The control objective is to design a closed loop law for the linear and angular velocities $\underline{u}$ and $\underline{\omega}$ such that the kinematic model of the underactuated vehicle is asymptotically driven to the origin of the $<a>$ frame along the $\underline{i}_a$ axis while the constraints given by equations (1) and (2) are satisfied.

The velocity vector field $\underline{v}_h$

When the vehicle is not located in the origin of the target frame $<a>$, i.e. $e = \|\underline{e}\| \neq 0$ and $\underline{r}_\theta$ is well defined, i.e. $\theta \neq 0, \pi$, the reference $<e>$ may be defined as

$< e >= \{\underline{i}_e, \underline{j}_e, \underline{k}_e\}$ where $\underline{k}_e \equiv \underline{r}_\theta$, $\underline{i}_e \equiv \underline{e}/\|\underline{e}\|$, $\underline{j}_e = \underline{r}_\theta \wedge \underline{i}_e$. The vector field $\underline{v}_h$ is defined to be such that the projection of $\underline{v}_h$ on the reference $<e>$ is given by

$$\begin{aligned} {}^e\underline{v}_h &= {}^e[\, \gamma_e\, \underline{e} + \gamma_\theta\, \theta\, (\underline{r}_\theta \wedge \underline{e})\,] = \\ &= \gamma_e\, e\underline{i}_e + \underline{j}_e \gamma_\theta\, \theta e \quad : \quad \gamma_e\, , \gamma_\theta\, > 0. \end{aligned} \tag{3}$$

Given the nature of reference $< e >$, the field $\underline{v}_h$ is discontinuous when $\theta = \pi$. This is actually a drawback of the proposed strategy as when $\theta \underline{r}_\theta$ is not defined neither equation (3) is. As a consequence the field $\underline{v}_h$ is not defined when $\theta = \pi$, while for $\theta = 0$ the vector $\theta \underline{r}_\theta$ is well defined and equal to the null vector so that $\theta = 0 \Rightarrow \underline{v}_h = \gamma_e\, e\underline{i}_e$. Notice that by construction a point subject to the velocity given by equation (3) would exponentially converge to the target provided that $e_{t=0} \neq 0$ and $\theta_{t=0} \in [0, \pi)$. The basic idea is that of building a closed loop control law for the angular velocity of the nonholonomic vehicle that asymptotically drives $\underline{i}_b$ parallel to $\underline{v}_h$.

The steering law

Given the above definitions of $\underline{v}_h$ and β a closed loop control law for the angular velocity $\underline{\omega}$ that drives β to zero is to be found. Consider the more general situation in which two unit vectors $\underline{\nu} \equiv \underline{v}_h/\|\underline{v}_h\|$ and $\underline{i}_b$ are given such that their angular velocities are $\underline{\omega}_\nu$ and $\underline{\omega}$ and a law for $\underline{\omega}$ is searched such that the vector $\underline{\beta} \equiv \underline{r}_\beta\, \beta$ is asymptotically null. This control problem may be approached considering the Lyapunov function

$$V_\beta = \frac{1}{2}\, \underline{\beta} \cdot \underline{\beta} = \frac{1}{2}\, \beta^2 \tag{4}$$

and its time derivative

$$\dot{V}_\beta = \underline{\beta} \cdot (\underline{r}_\beta \dot{\beta} + \dot{\underline{r}}_\beta \beta) = \underline{\beta} \cdot \underline{r}_\beta \dot{\beta} \tag{5}$$

as $\dot{\underline{r}}_\beta = \underline{\omega} \wedge \underline{r}_\beta \;\perp\; \underline{\beta}$. By standard kinematic it follows that

$$\dot{\beta} = \underline{r}_\beta \cdot (\underline{\omega}_\nu - \underline{\omega}) \tag{6}$$

so that in order to guarantee that equation (5) is negative definite it is sufficient, by example, that

$$\underline{r}_\beta \left(\underline{r}_\beta \cdot (\underline{\omega}_\nu - \underline{\omega}) \right) = -k\, \underline{\beta} \;:\; k > 0. \tag{7}$$

By direct calculation it is found that equation (7) is satisfied choosing

$$\underline{\omega} = k\, \underline{\beta} + \underline{r}_\beta (\underline{r}_\beta \cdot \underline{\omega}_\nu) \tag{8}$$

which guarantees exponential convergence of β to zero as can be seen replacing (8) in (6). Notice that the angular velocity $\underline{\omega}_\nu$ of a unit vector $\underline{\nu}$ is always given by $\underline{\omega}_\nu = \underline{\nu} \wedge \dot{\underline{\nu}}$ so that finally equation (8) may be expressed as

$$\underline{\omega} = k\, \underline{\beta} + \underline{r}_\beta \left(\underline{r}_\beta \cdot (\underline{\nu} \wedge \dot{\underline{\nu}}) \right). \tag{9}$$

The control signal given by equation (9) satisfies the constraint (2), but may be not defined in correspondence of those states in which $\underline{r}_\beta$ or $\underline{\nu}$ are not defined. These cases will be more deeply discussed in a following section.

The time derivative of $\underline{\nu}$

In order to implement equation (9) given that $\underline{\nu}$ is the unit vector parallel to $\underline{v}_h$, its time derivative $\underline{\dot{\nu}}$ must be explicitly computed. With reference to figure (1) and to the definition (3) of $\underline{v}_h$, $\underline{\dot{\nu}}$ may be generated at most by three different kinds of infinitesimal motions of the frame $<b>$, i.e. $\underline{\dot{\nu}} = \underline{\dot{\nu}}_1 + \underline{\dot{\nu}}_2 + \underline{\dot{\nu}}_3$:

- $\underline{\dot{\nu}}_1$ due to linear movements of $\underline{e}$ along $\underline{i}_e$
- $\underline{\dot{\nu}}_2$ due to rotations of $\underline{e}$ around $\underline{r}_\theta$
- $\underline{\dot{\nu}}_3$ due to rotations of $\underline{e}$ around $\underline{j}_e = \underline{r}_\theta \wedge \underline{i}_e$.

Given that both components of $\underline{v}_h$ are linear in e, $\underline{\dot{\nu}}_1 = 0$ as $\underline{\nu}$ is actually *not* affected by the first kind of infinitesimal motions, namely translations along $\underline{i}_e$ leave $\underline{\nu}$ unchanged. As far as the rotations around $\underline{r}_\theta$ are concerned it follows that

$$\underline{\dot{\nu}}_2 = (\dot{\delta} + \dot{\theta})\,(\underline{r}_\theta \wedge \underline{\nu})$$

being δ the angle between $\underline{i}_e$ and $\underline{v}_h$ as shown in figure (1). The time derivative of δ can be evaluated noticing that:

$$\begin{aligned} {}^e\underline{e} \cdot {}^e\underline{v}_h &\equiv \|\underline{v}_h\|\|\underline{e}\| \cos\delta = e^2\sqrt{(\gamma_e^2 + \gamma_\theta^2\,\theta^2)}\ \ \cos\delta = \gamma_e\ e^2 && (10)\\ {}^e\underline{e} \wedge {}^e\underline{v}_h &\equiv {}^e\underline{r}_\delta \|\underline{v}_h\|\|\underline{e}\| \sin\delta = \\ &= {}^e\underline{r}_\delta\, e^2\sqrt{(\gamma_e^2 + \gamma_\theta^2\,\theta^2)}\ \ \sin\delta = {}^e\underline{r}_\delta\ \gamma_\theta\ \ \theta e^2, && (11) \end{aligned}$$

from which it follows that $\cos\delta$ and $\sin\delta$ are given by

$$\cos\delta = \frac{\gamma_e}{\sqrt{(\gamma_e^2 + \gamma_\theta^2\,\theta^2)}} \tag{12}$$

$$\sin\delta = \frac{\gamma_\theta\ \theta}{\sqrt{(\gamma_e^2 + \gamma_\theta^2\,\theta^2)}}. \tag{13}$$

Finally differentiating equation (12) and using equation (13) the time derivative of δ is found to be

$$\dot{\delta} = \frac{\gamma_e\ \gamma_\theta}{\gamma_e^2 + \gamma_\theta^2\,\theta^2}\,\dot{\theta}. \tag{14}$$

Assuming all the relevant unit vectors ($\underline{r}_\theta$, $\underline{r}_\alpha$, $\underline{\nu}$) to be well defined, the value of $\dot{\theta}$ can be computed noticing that θ changes if and only if there is an angular velocity parallel to $\underline{r}_\theta$ applied on $\underline{e}$. The only motion that can generate such angular velocity is the linear translation along $\underline{i}_b$ which, in fact, causes a rotation of $\underline{e}$ having an angular velocity $\underline{\omega}_e = \underline{r}_\alpha(u/e)\,\sin\alpha$. As a consequence of this angular velocity it follows that $\dot{\theta} = \underline{r}_\theta \cdot \underline{r}_\alpha(u/e)\,\sin\alpha$ and thus

$$\underline{\dot{\nu}}_2 = \frac{u\sin\alpha}{e}\left(1 + \frac{\gamma_e\ \gamma_\theta}{\gamma_e^2 + \gamma_\theta^2\,\theta^2}\right)(\underline{r}_\theta \cdot \underline{r}_\alpha)(\underline{r}_\theta \wedge \underline{\nu}). \tag{15}$$

As far as the contribution $\underline{\dot{\nu}}_3$ to $\underline{\dot{\nu}}$ due to rotations around $\underline{j}_e = \underline{r}_\theta \wedge \underline{i}_e$ is concerned it must first be noticed that such motions do not affect the value of θ. This can be shown, by example, considering the effect of a constant rotation around $\underline{j}_e$, i.e. an angular velocity $\underline{\sigma} = \sigma \underline{j}_e$: the absolute time derivative of $\underline{i}_e$ due to such angular velocity would be given by

$$\frac{d_{<a>}}{dt}\underline{i}_e = \underline{\sigma} \wedge \underline{i}_e = \sigma \underline{j}_e \wedge \underline{i}_e = -\sigma \underline{r}_\theta$$

and as the absolute derivative of the fixed reference unit vectors is identically null ($\frac{d_{<a>}}{dt}\underline{i}_a = 0$) it follows that

$$\frac{d_{<a>}}{dt}\cos\theta = \frac{d_{<a>}}{dt}(\underline{i}_a \cdot \underline{i}_e) = -\sigma\, \underline{i}_a \cdot \underline{r}_\theta = 0$$

showing that the angle θ does *not* change in consequence of rotations of $\underline{e}$ around $\underline{j}_e$. This fact allows to compute $\underline{\dot{\nu}}_3$ as follows:

$$\underline{\dot{\nu}}_3 = \frac{\underline{u} \cdot \underline{r}_\theta}{e}\left(\underline{j}_e \wedge \underline{\nu}\right) \tag{16}$$

being $\underline{u} \cdot \underline{r}_\theta$ the only component of the systems velocity that may cause a rotation of $\underline{e}$ around $\underline{j}_e$. Recalling the above partial results, the time derivative of $\underline{\nu}$ is found to be:

$$\underline{\dot{\nu}} = \frac{u}{e}\left(\left(1 + \frac{\gamma_e\, \gamma_\theta}{\gamma_e^2\, + \gamma_\theta^2\, \theta^2}\right)(\underline{r}_\theta \cdot \underline{r}_\alpha \sin\alpha)\underline{r}_\theta + (\underline{i}_b \cdot \underline{r}_\theta)\,(\underline{r}_\theta \wedge \underline{i}_e)\right) \wedge \underline{\nu}. \tag{17}$$

Once again it is observed that this equation is defined only when the relevant unit vectors are well defined. Replacing equation (17) in (9) a closed loop equation for $\underline{\omega}$ is found that drives the vehicles axis $\underline{i}_b$ asymptotically parallel to the field $\underline{v}_h$. It is a straightforward exercise to show that in the planar case, i.e. when $\underline{r}_\alpha = \underline{r}_\theta = \underline{r}_\beta = \underline{k}_a$, the resulting angular velocity $\underline{\omega}$ is exactly the same one obtained for the $2D$ unicycle model [6] applying the same position control strategy.

Discontinuities in the steering law

By direct analysis of equations (9) and (17) it is apparent that the steering control law may not be defined if $\beta = \pi$, $\beta = 0$, $\theta = \pi$ or $\theta = 0$ while the cases $\alpha = 0, \pi$ should not worry as the term $\underline{r}_\alpha \sin\alpha$ in equation (17) is always well defined. The most serious case is perhaps $\theta = \pi$ as in this situation the field $\underline{v}_h$ itself is not defined and so neither is the scalar β. As a consequence the steering control $\underline{\omega}$ will have to be discontinuous when $\theta = \pi$. A possible choice for $\underline{\omega}_{|\theta=\pi}$ is

$$\underline{\omega}_{|\theta=\pi} = \begin{cases} 0 & \text{if } \alpha \notin \{0, \pi\} \\ \varepsilon\, \underline{k}_b & \varepsilon > 0 \text{ otherwise.} \end{cases} \tag{18}$$

Although without entering in the details of the analysis, it is worthwhile to examine at least qualitatively the consequences of this choice. Assuming that the vehicle moves in only one forward direction as will be apparent from the next section, the case $\theta = \pi \,\cup\, \alpha \in \{0, \pi\}$ corresponds to the situation in which the vehicle is moving along the positive part of the $\underline{i}_a$ axis of the target frame $<a>$ towards the target ($\alpha = 0$) or away from it ($\alpha = \pi$). In such situation any infinitesimal angular velocity driving the vehicle off the $\underline{i}_a$ axis as suggested by equation (18) will bring the system in a region where all relevant angles and unit vectors are properly defined for the implementation of equations (9) and (17). Moreover notice that due to the radial symmetry of the $\underline{v}_h$ field in any neighborhood of the $\underline{i}_a$ axis and due to the exponential convergence of β given by (9), the implementation of equation (18) when $\theta = \pi \,\cup\, \alpha \in \{0, \pi\}$ suggests that the $\underline{i}_a$ axis will not be crossed again once it is left. A similar reasoning justifies the choice $\underline{\omega}_{|\theta=\pi} = 0$ if $\alpha \notin \{0, \pi\}$ as in this situation the "singular" axis $\underline{i}_a$ is left behind by just moving in the given $\underline{i}_b$ direction. The case $\theta = 0$ is less problematic as actually in such case $\underline{v}_h$ as given by equation (3) is well defined and parallel to $\underline{i}_a$. Notice that when $\theta = 0$ the angles α and β are equivalent, i.e. $(\alpha_{|\theta=0}) \equiv (\beta_{|\theta=0})$ and $\underline{\nu}_{|\theta=0} \equiv \underline{i}_e$. A possible choice for $\underline{\omega}_{|\theta=0}$ is

$$\underline{\omega}_{|\theta=0} = \begin{cases} (k\,\beta + (u/e)\sin\alpha)\underline{r}_\beta & \text{if } \beta \notin \{0, \pi\} \\ \varepsilon\,\underline{k}_b : \varepsilon > 0 & \text{if } \beta = \pi \\ 0 & \text{if } \beta = 0. \end{cases} \tag{19}$$

The first of the three cases given by equation (19) is obtained from equation (9) noticing that if $\underline{\nu} = \underline{i}_e$ as when $\theta = 0$, then $\dot{\underline{\nu}}_{|\theta=0} = (\underline{r}_\alpha \wedge \underline{i}_e)(u/e)\sin\alpha$. This specific choice of $\underline{\omega}_{|\theta=0}$ when $\beta \notin \{0, \pi\}$ guarantees that the Lyapunov function (4) and its time derivative (5) are continuous. The choice $\underline{\omega}_{|\theta=0} = \varepsilon\,\underline{k}_b : \varepsilon > 0$ when $\beta = \pi$ follows from the observation that in such situation the vehicle is moving along the negative part of the $\underline{i}_a$ axis in the opposite direction of the target. Any infinitesimal angular velocity will drive the system off this axis in a region where all relevant angles and unit vectors are suitably defined for the implementation of equations (9) and (17). Moreover the structure of the field $\underline{v}_h$ suggests that once this happens the vehicle will not cross the $\underline{i}_a$ axis with $\beta = \pi$ again. The last case $\theta = \beta = \alpha = 0$ corresponds to the trivial situation in which the vehicle is moving on the $\underline{i}_a$ axis towards the target and thus it may simply proceed on a straight line. Finally it should be noticed that thanks to the exponential convergence of β to zero the unit vector $\underline{r}_\beta$ used to compute the angular velocity $\underline{\omega}$ will be actually well defined at all finite times as long as $\beta_{|t=0} \neq 0, \pi$. This suggests that in the case that the initial configuration of the vehicle should imply $\beta = 0$ or $\beta = \pi$ any angular velocity satisfying equation (2) may be applied at the time $t = 0$ in order to enter the region of the state space where equations (9) and (17) may be directly implemented.

The linear velocity law

Having computed a steering law that guarantees asymptotic convergence of β to zero, the most natural choice for the linear velocity is

$$u = \|\underline{v}_h\| = e\sqrt{\gamma_e^2 + \gamma_\theta^2\,\theta^2}\,. \tag{20}$$

With such law for u the vehicle will asymptotically follow the velocity field $\underline{v}_h$ with a well defined angular velocity as the ratio u/e required in equation (17) is defined in the whole state space at all finite times. This follows from the observation that the state equation for e is given by

$$\dot{e} = -u\cos\alpha \tag{21}$$

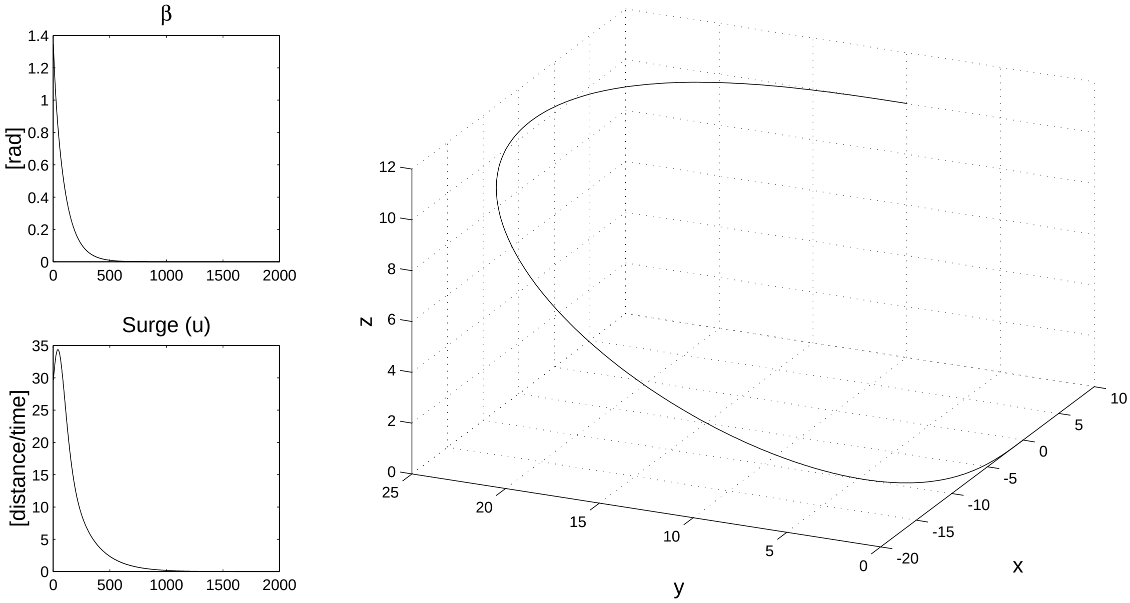

Figure 2: Path resulting from the application of the proposed closed loop control strategy. Refer to the text for greater details regarding this example.

thus replacing equation (20) in (21) it is found that

$$e(t) = e_{|t=0} \exp\left[-\int_0^t \sqrt{\gamma_e^2 + \gamma_\theta^2\,\theta(\tau)^2}\,\cos\alpha(\tau) d\tau\right] \tag{22}$$

showing that if $e_{|t=0} \neq 0$ then e is strictly positive at *all* finite times. Indeed the application of the proposed strategy guarantees that e tends to zero asymptotically -actually exponentially- and at the same time it guarantees that the singular point $e = 0$ where α and θ are not defined is never touched during the closed-loop time evolution of the systems. Moreover having the linear velocity given by equation (20) constant sign it guarantees the absence of cusps in the vehicles path thus satisfying a major requirement for real world applications.

SIMULATIONS AND CONCLUSIONS

The above presented approach has been tested by simulations one of which is here reported for reference. In figure (2) the path resulting from initial position $(x_0, y_0, z_0) = (10, 10, 10)$ and orientation (roll, pitch, yaw) $(\phi_0, \theta_0, \psi_0) = (0, 0, \pi/2)$ with gains $k = 1$, $\gamma_e = 1/2$ and $\gamma_\theta = 3/4$ is reported together with the time history of the surge and of the variable β that converges exponentially as expected. The pitch and yaw velocities computed with the proposed closed loop law relative to this case are plotted in figure (3). It has been shown that adopting a polar like description of a 3D vehicle it is possible to design a closed loop control law having only isolated discontinuity points that drives the position configuration to zero. Strictly speaking the proposed solution does not solve the point stabilization problem for the 3D nonholonomic systems as the target configuration ($e = 0$) lies on the boundary of the polar-like state space domain, but is not part of it. Nevertheless this has virtually no importance from the practical point of view as the proposed strategy can

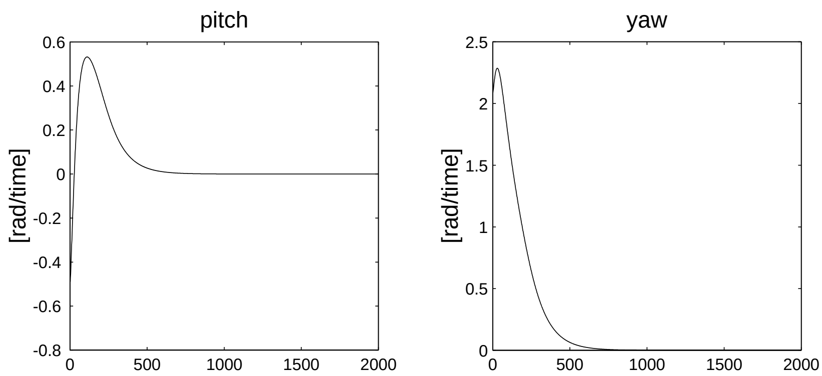

Figure 3: Time history of the yaw and pitch variables relative to the path displayed in figure (2).

be applied from *any* neighborhood of the null measure set $e = 0$. The use of polar-like variables to design time-invariant controllers for mechanical systems that would otherwise suffer from Brocketts negative result is not limited in the literature to the unicycle model but has been recently adopted [7] to control a planar space robot with a two link manipulator. Because of the presence of a null measure set (containing the equilibrium) from which these control laws fail to converge they are sometimes called "almost" smooth. The proposed control design methodology exploits the prior definition of a velocity vector field that would exponentially drive the configuration error of a nonholonomic constraint-free systems to zero. It is actually sufficient to define a steering law that asymptotically orients the nonholonomic vehicle along the direction of this field to achieve the control objective.

REFERENCES

1. A. Astolfi, "Exponential stabilization of a wheeled mobile robot via discontinuous control," *ASME Jou. of Dyn. Sys. Mea. and Cont.*, March 1999.
2. M. Aicardi, G. Casalino, A. Bicchi, and A. Balestrino, "Closed loop steering of unicycle-like vehicles via lyapunov techniques," *IEEE Robotics and Automation Magazine*, pp. 27–35, March 1995.
3. G. Indiveri, "Kinematic time-invariant control of a $2D$ nonholonomic vehicle," in *38th IEEE Conference on Decision and Control CDC'99*, (Phoenix, USA), December 1999.
4. M. Aicardi, G. Cannata, and G. Casalino, "Smoothness of a feedback control law for a non-holonomic 3D vehicle," in *IFAC 3rd Int. Workshop on Motion Control*, (Grenoble, France), pp. 239–244, September 21-23 1998.
5. O. Egeland, M. Dalsmo, and O. J. Sørdalen, "Feedback control of a nonholonomic underwater vehicle with a constant desired configuration," *Int. Jou. of Robotics Research*, vol. 15, no. 1, pp. 24–35, 1996.
6. M. Aicardi, G. Cannata, G. Casalino, and G. Indiveri, "On the stabilization of the unicycle model projecting a holonomic solution," in *8th Int. Symposium on Robotics with Applications, ISORA 2000*, (Maui, Hawaii, USA), June 11-16 2000.
7. R. Mukherjee, M. Kamon,"Almost sooth time-invariant control of planar space multibody systems," *IEEE Trans. on Robotics and Automation*, vol. 15, no. 2, pp. 268-280, April 1999.