On a closed loop time invariant position control solution for an underactuated 3D underwater vehicle: implementation, stability and robustness considerations.

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Abstract— **Two discontinuous solutions for the kinematic position and attitude closed loop control problem of an underactuated floating body are considered. The first is derived on the basis of Lyapunovs stability theory while the second is designed exploiting a novel idea: first a vector field is defined such that an ideal point free to move in any direction would exponentially converge to the desired configuration and then a steering law for the underactuated vehicle is derived such that it is exponentially parallel to the above mentioned field. Both solutions yield cusp-free and asymptotically null curvature paths which is a major practical constraint in real world applications.**

I. Introduction

The feedback control of a nonholonomic 3D floating vehicle is considered: namely the control objective is to drive a vehicle moving in 3D space to a given point and heading along a given line having as control inputs a 1D linear velocity (surge velocity) and a 2D angular one allowing the vehicle to rotate around any axis normal to the surge one. This kind of kinematic describes a large class of floating systems including underwater or space vehicles. Brocketts theorem [1] prevents the synthesis of a globally convergent smooth pure feedback control law for such kind of systems and indeed time varying [2][3] or discontinuous [4] closed loop solutions have been suggested in the literature. In spite of their mathematical elegance time-varying solutions yield quite complex controllers and the resulting paths may present cusps due to the fact that the vehicle is supposed to move in both the forward and backward directions, while the great majority of autonomous underwater vehicles (AUVs) travel in the only positive surge direction. This is indeed a major point as even time-invariant solutions [5] are affected by this problem.

As suggested by Brockett himself [1] a suitable (i.e. singular) choice of state variables, e.g. a polar-like set of coordinates for the unicycle model, may transform a given system subject to Brocketts negative result into one for which such theorem does not prevent the existence of a pure feedback control law. It has been shown that the application of this idea to the unicycle model allows to easily determine a simple nonlinear time-invariant closed loop law that drives the configuration error to zero as long as the vehicles is not initially positioned in the target point where the coordinate transformation singularity takes place [6][7]. The idea of mapping the systems by a non global diffeomorphism state transformation such that the transformed system is not subject to Brocketts negative result is not limited in the literature to the 2D or 3D unicycle kinematic model [6][7] [8][9][4] [10], but has been applied to several other nonholonomic systems with success [11][12]. Recent results [13] show that with such kind of strategy it is indeed possible to determine time-invariant, cusp free solutions that guarantee convergence to the target configuration as long as the vehicle is not initially positioned in the target point. From a strictly technical point of view the solutions provided by this methodology are not stabilizing ones as the equilibrium state belongs only to the boundary of the domain of definition of the transformed state variables. Nevertheless from a practical point of view this has actually no importance as the proposed steering laws may be applied to drive the vehicle to the target from any initial configuration as long as the starting position differs from the target one.

The present paper focuses on a brief analysis of the implementation requirements of these laws, and of the stability and robustness properties that these solutions could guarantee in real world applications.

II. The model

With reference to figure (1) consider a rigid body having a body fixed reference $<b> = \{\underline{i}_b, \underline{j}_b, \underline{k}_b\}$ such that the linear $\underline{u}$ and angular $\underline{\omega}$ velocities obey

$$\underline{u} = u\,\underline{i}_b \tag{1}$$

$$\underline{\omega} \cdot \underline{i}_b = 0 \tag{2}$$

being $\cdot$ the scalar product operator and u the euclidean norm $\| \; \|$ of $\underline{u}$. The vector $\underline{e} = e\underline{i}_e : \|\underline{i}_e\| = 1$ is defined to point from $<b>$ to the origin of the target fixed reference $<a> = \{\underline{i}_a, \underline{j}_a, \underline{k}_a\}$. The angles between $\underline{i}_a$, $\underline{i}_b$ and $\underline{e}$ are θ and α such that if they are different from multiples of π then the unit vectors $\underline{r}_\theta = \underline{i}_a \wedge \underline{i}_e / \|\underline{i}_a \wedge \underline{i}_e\|$ and $\underline{r}_\alpha = \underline{i}_b \wedge \underline{i}_e / \|\underline{i}_b \wedge \underline{i}_e\|$ the $\wedge$ indicating the standard vector product, are well defined. The position control problem consists then in finding $u(\underline{e}, \underline{\theta}, \underline{\alpha})$ and $\underline{\omega}(\underline{e}, \underline{\theta}, \underline{\alpha})$ such that asymptotically $(\underline{e}, \underline{\theta}, \underline{\alpha}) \rightarrow (\underline{0}, \underline{0}, \underline{0})$ being $\underline{\theta} = \theta\,\underline{r}_\theta$ and $\underline{\alpha} = \alpha\,\underline{r}_\alpha \;\forall\; \theta, \alpha \neq n\pi : n \in Z$. The control objective may be actually described stating that the scalars (e, θ, α) are requested to converge to zero, but as their derivatives explicitly depend on the unit vectors $\underline{i}_e$, $\underline{r}_\theta$ and $\underline{r}_\alpha$ [4] the controls u and $\underline{\omega}$ will be functions of $(\underline{e}, \underline{\theta}, \underline{\alpha})$ rather than of their norms. As a consequence of the singularities occurring in the definition of $\underline{r}_\theta$ and $\underline{r}_\alpha$ when α or θ are multiples of π the control laws $u(\underline{e}, \underline{\theta}, \underline{\alpha})$ and $\underline{\omega}(\underline{e}, \underline{\theta}, \underline{\alpha})$ may present discontinuities.

III. Two possible solutions

Given the above polar-like kinematic model and position control problem statement, two different solutions have been recently proposed [13][14] both providing cusp-free paths, but affected by isolated discontinuity points. The first is based on the following idea: first a velocity vector field $\underline{v}_h = \gamma_e\, e\underline{i}_e + \underline{j}_e \gamma_\theta\, \theta e : \gamma_e\,, \gamma_\theta\; > 0$ is defined such that if the origin of the frame $<b>$ was free to move with no non-holonomic constraint, moving with such velocity it would globally and exponentially converge to the target along the desired direction. Then a steering law for $\underline{\omega}$ is computed such that $\underline{i}_b$ is globally and exponentially parallel to $\underline{v}_h$ while the velocity of advance u of the vehicle is chosen to be $\|\underline{v}_h\|$. The application of the same idea to the planar case is described in [9]. When all the suitable unit vectors involved are well defined, the resulting control laws turn out to be

$$\underline{\omega} = k\,\underline{\beta} + \underline{r}_\beta \left(\underline{r}_\beta \cdot (\underline{\nu} \wedge \dot{\underline{\nu}})\right) : k > 0 \tag{3}$$

$$\begin{aligned} \dot{\underline{\nu}} &= \frac{u}{e}\left(\left(\left(1 + \frac{\gamma_e\,\gamma_\theta}{\gamma_e^2 + \gamma_\theta^2\,\theta^2}\right)(\underline{r}_\theta \cdot \underline{r}_\alpha \sin\alpha)\underline{r}_\theta + \right.\right. \\ &+ \left.\left. (\underline{i}_b \cdot \underline{r}_\theta)\,(\underline{r}_\theta \wedge \underline{i}_e)\right) \wedge \underline{\nu}\right) \end{aligned} \tag{4}$$

$$u = \|\underline{v}_h\| = e\sqrt{\gamma_e^2 + \gamma_\theta^2\,\theta^2} \tag{5}$$

being $\underline{\nu}$ the unit vector parallel to $\underline{v}_h$, i.e. $\underline{\nu} = \underline{v}_h / \|\underline{v}_h\|$, β the angle between $\underline{i}_b$ and $\underline{v}_h$ and $\underline{r}_\beta = \underline{i}_b \wedge \underline{\nu} / \|\underline{i}_b \wedge \underline{\nu}\|$ the unit vector normal to the plane $\{\underline{i}_b, \underline{v}_h\}$ when $\beta \neq n\pi : n \in Z$. For a detailed description of this control solution refer to [13] where the convergence and stability properties are addressed together with the discontinuities and their effect on the overall system performance.

The second solution builds on the results presented in [4] and is described in detail in [14]. The main idea consists in deriving a steering law that guarantees the candidate Lyapunov function

$$V = 1/2(\alpha^2 + h\theta^2) : h > 0 \tag{6}$$

to have a negative definite time derivative and the configuration error $(\underline{e}, \underline{\theta}, \underline{\alpha})$ to asymptotically converge to zero. This is achieved choosing $u = \gamma_e\, e : \gamma_e \; > 0$ that being always positive guarantees the absence of cusps from the paths. Denoting with ${}^c\underline{x}$ the projection of the vector $\underline{x}$ on reference $<c>$, the complete solution [14] consists of the controls:

$$u = \gamma_e\; e \;\; ; \;\; \gamma_e > 0 \tag{7}$$

$${}^b\underline{\omega} = K\,{}^b\underline{\alpha} + \gamma_e\,({}^b\underline{\alpha} + h\,{}^b\underline{\tilde{\theta}})\,\frac{\sin\alpha}{\alpha} \;\; ; \;\; K, h > 0 \tag{8}$$

$${}^b\underline{\tilde{\theta}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \underline{\theta} \tag{9}$$

that, as for the other solution, may present discontinuities when the relevant unit vectors fail to be properly defined.

If the position control task should degenerate in the one of reaching the origin of frame $<a>$ regardless of the direction of arrival it would be sufficient to choose $\gamma_\theta = 0$ and $h = 0$ in the two solutions. Lengthy, but straightforward direct calculations show that in such case the two solutions are equivalent, as can be intuitively understood from figure (1). In particular both solutions would guarantee exponential convergence of α to zero with $u = \gamma_e\, e$.

IV. Implementation considerations

If on the one hand it can be claimed that neglecting the vehicles dynamics may cause poor performance of the controller, it must also be observed that the estimation of the systems velocity vector, required to implement any solution taking the dynamics into account, is by no means trivial or even possible for certain classes of underwater vehicles. On the contrary the suggested solutions may indeed by implemented measuring "only" the scalar distance from the target (e) and the vehicles attitude from which all the required angular variables may be computed. Of course the measurement and/or estimation of these quantities will depend on the available sensors of the specific vehicle.

As far as the stability of the proposed solutions is concerned it should be noticed that the point stabilization problem is actually not solved in the strict sense as the equilibrium ($e = 0$) is not part of the domain of definition of the control law, but lies on its boundary. This is an obvious consequence of the use of polar-like variables but it is not a problem from the practical point of view: indeed the configuration error is driven to zero from any initial configuration such that $e|_{t=0} \neq 0$. Notice moreover that not only the paths are cusp-free as shown previously, but with a suitable choice of the controller gains it is always possible to guarantee that the target is approached on a null curvature path. In particular it is necessary to select the gains such that the angular variables converge to zero faster than e (see [13] for details). These fundamental practical constraints are not easily satisfied within the time-varying control framework.

The major price to be paid for the nice properties above reported is the presence of discontinuities in the steering law. A detailed description of the configurations in which these occur and of a possible way to deal with them is found in [4][13]. Notice however that the singularities occurring when $e \neq 0$ and some of the necessary unit vectors are not defined may be regarded as "isolated", in the sense that the corresponding configurations are unstable in closed loop and the suggested control laws are well defined in any neighborhood of those configurations. This is why with a suitable definition of the steering law in correspondence of those singularities, in spite of introducing a discontinuity point, these configurations are left behind and the overall error may still converge to zero.

The situation is different in a neighborhood of $e = 0$. In the ideal case of no measurement noise the scalar e will tend to zero continuously and exponentially and thus both θ and $\underline{r}_\theta$ will asymptotically be continuous time functions. In the more realistic case of imperfect state measurement the angle θ will "jump" (see figure 1) from $\theta \approx 0$ to $\theta \approx \pi$ when e is in a neighborhood of the origin and affected by measurement noise. This of course may cause a high frequency discontinuous steering control action similar to what occurs with sliding mode discontinuous controllers. This phenomenon may be limited introducing a deadband zone, i.e. a small set in the state space containing $e = 0$, in which the steering input is put to zero. The size and topology of such set must be tuned according to the state measurement noise. From a practical point of view this does not seem to be a serious problem as the application of the suggested controllers will still guarantee to make the configuration error smaller or equal to a threshold depending on the precision of the state estimation and measurement techniques employed.

V. Conclusions

The proposed control strategies have been tested with some simulations here described. First the controller given by equations (7) and (8) has been adopted to steer to the origin a systems being initially positioned in $x_0 = 5, y_0 = 5, z_0 = 5$ and having initial attitude $\phi_0 = 0, \theta_0 = 0, \psi_0 = 0$. The gain parameters were fixed to $K = 2.5, h = 2, \gamma_e = 1$. The resulting path is shown in figure (2) and the control inputs together with the decreasing Lyapunov function (6) in figure (3).

The controller given by equations (3) and (5) has been tested for the same initial configuration with gains fixed to $k = 1, \gamma_e = 1/2, \gamma_\theta = 0.7$. The resulting path is reported in figure (4) while the control inputs and the function V given by equation (6) are shown in figure (5).

Two kinematic discontinuous closed loop controllers to drive an underactuated underwater vehicle in a given point with a given orientation have been presented. The implementation of these laws requires the only knowledge of the distance from the target and the attitude of the vehicle. Notice that similar purely kinematic solutions for the planar case were most successfully implemented on the underwater vehicles Roby2 and Romeo of CNR-IAN, Genova, Italy [15](and references therein). On the basis of these promising results it can be expected that also the here suggested 3D extensions will behave fine when tested. The drawbacks and advantages of the proposed solutions with respect to other existing ones have been outlined and discussed.

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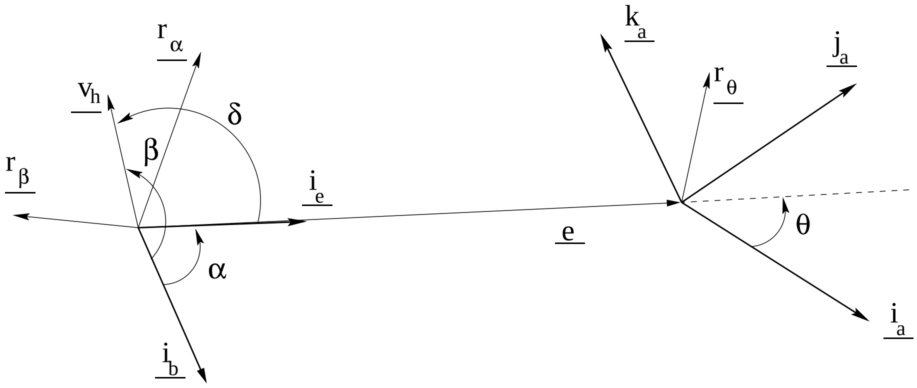

Fig. 1. Kinematic polar-like model

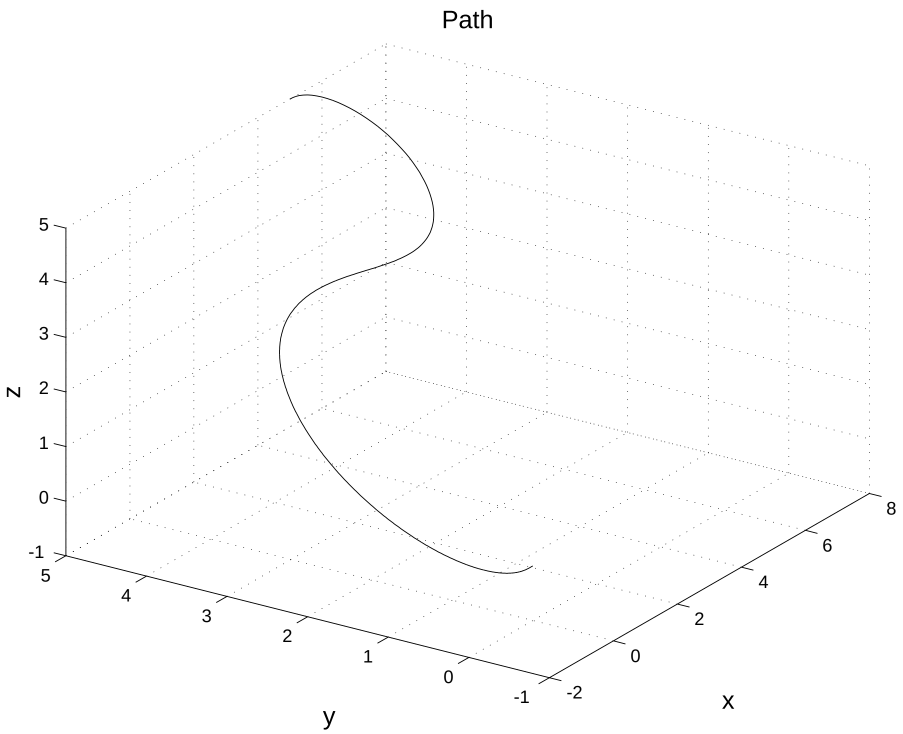

Fig. 2. Path resulting from the application of the controller given by equations (7) and (8) designed such that the Lyapunov function (6) has negative definite time derivative.

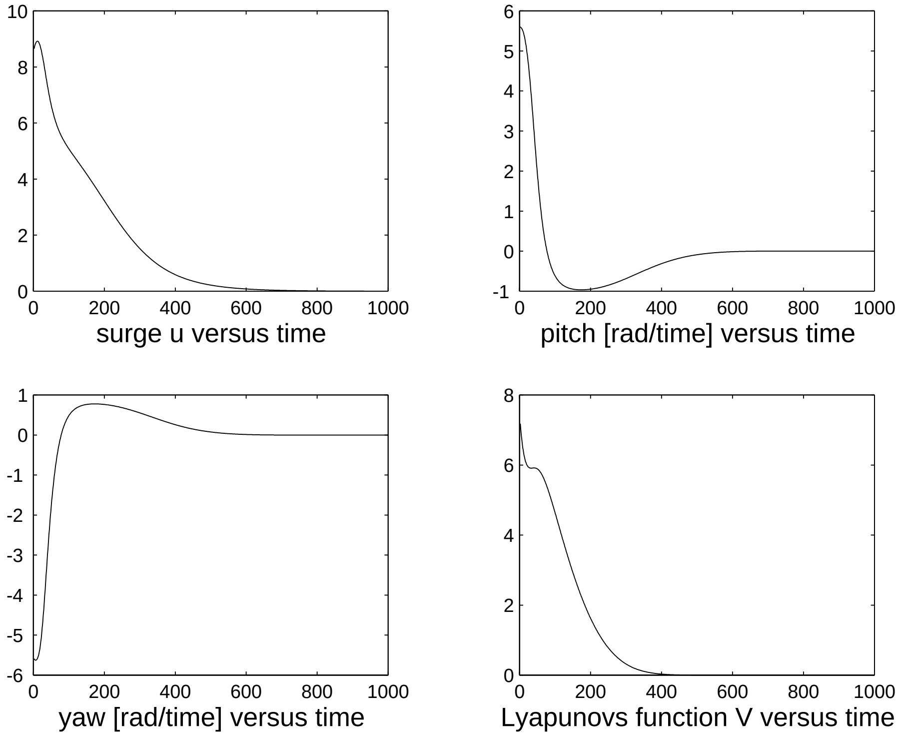

Fig. 3. Control inputs and Lyapunov function (6) relative to the path reported in figure (2). Refer to the text for details.

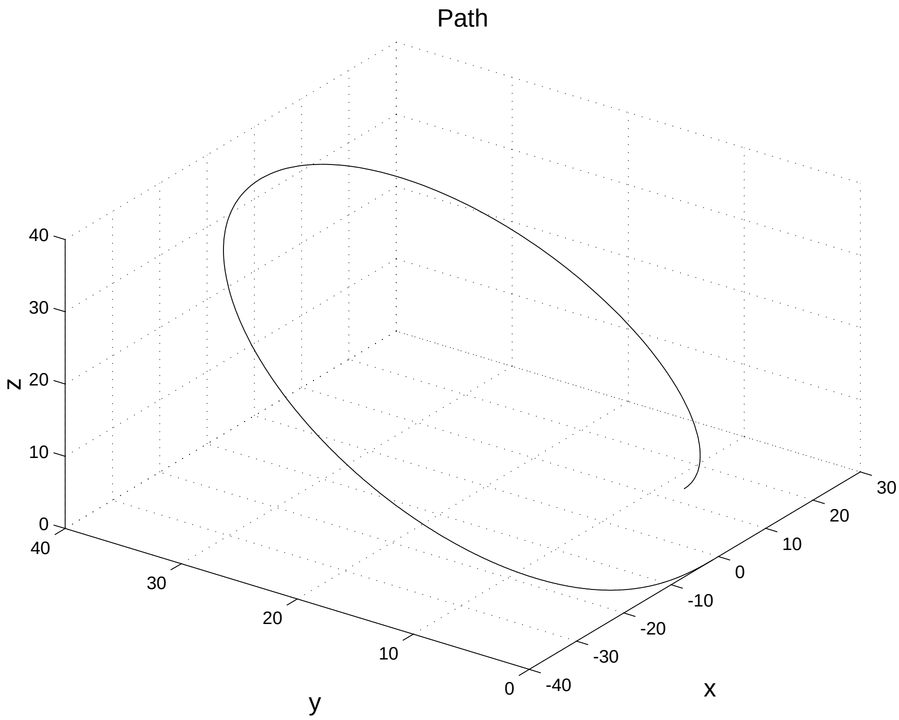

Fig. 4. Path resulting from the application of the controller given by equations (3) and (5) designed such that the $\beta \to 0$.

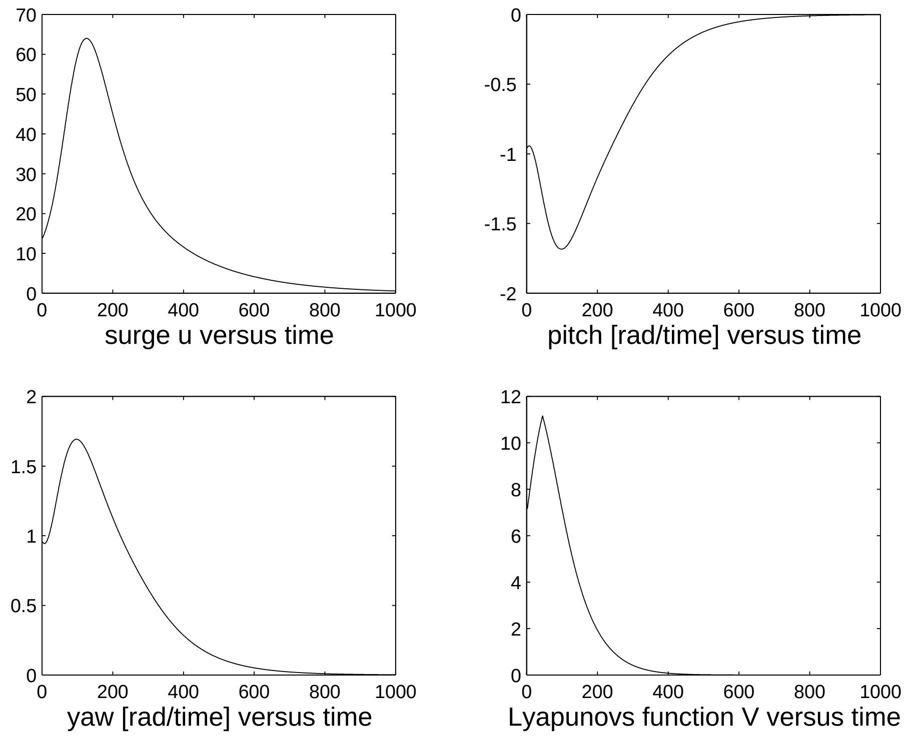

Fig. 5. Control inputs and Lyapunov function (6) relative to the path reported in figure (4). Notice that V is *not* always decreasing.

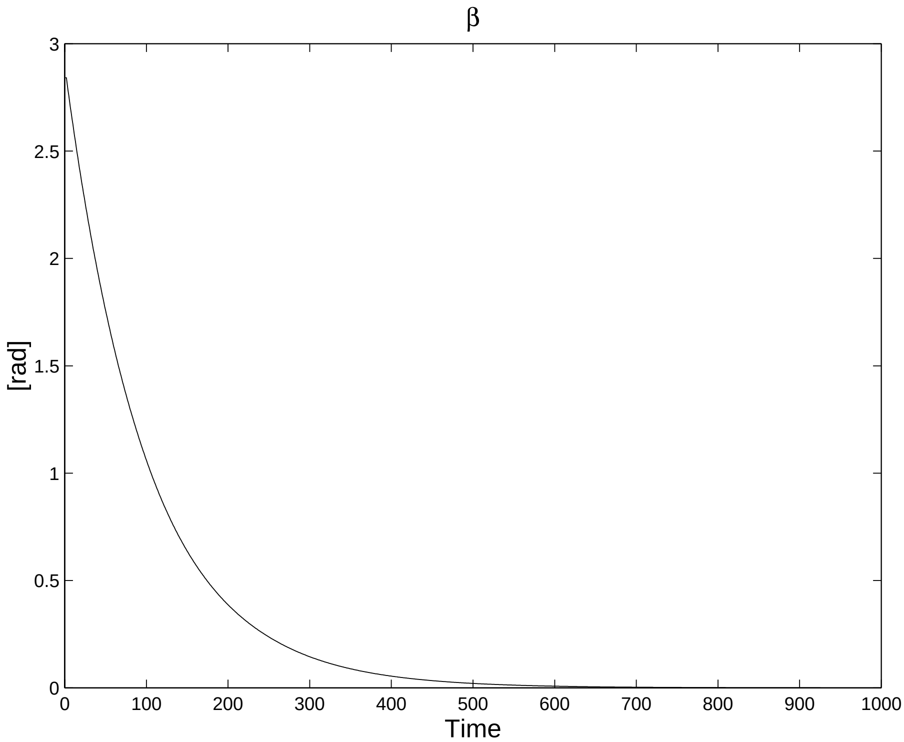

Fig. 6. Time history of the exponentially decreasing angle β relative to the path shown in figure 4.