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NONLINEAR TIME-INVARIANT FEEDBACK CONTROL OF AN UNDERACTUATED MARINE VEHICLE ALONG A STRAIGHT COURSE

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Abstract: A nonlinear, closed-loop, time-invariant controller that globally stabilizes an underactuated marine vehicle on a straight course is proposed. Traditional surface vessel linear course tracking autopilots are designed applying linear control methods on the linearized model, thus yielding only local results. Indeed due to the underactuated nature of the system, the model cannot be feedback linearized thus the most common and perhaps powerful tools of nonlinear control theory are ruled out and an alternative design strategy must be considered. At present no other smooth, time-invariant controller globally achieving the control objective is known to the authors. The overall convergence, stability and robustness to environmental disturbances is addressed and simulations are provided to show the controllers behaviour.

Keywords: Marine systems, nonlinear control, tracking.

1. INTRODUCTION

Consider a marine vehicle moving in the horizontal plane having only the surge and yaw axis directly actuated, while the sway is not: this is by far the most common configuration among marine systems. The issue of controlling such class of marine vehicles along a linear course while traveling at fixed constant speed and in spite of environmental disturbances has a great practical relevance. Indeed long range navigation tasks are frequently carried out traveling among via points on straight paths at fixed cruise speed. Traditional autopilots (Fossen 1994) are designed on the basis of a linearized dynamic model and are intended to stabilize the heading only while neglecting the non-actuated sway axis. The more advanced time-invariant nonlinear solutions proposed in the literature either fail to control all the three degrees of freedom in closed loop (Berge *et al.* 1998) or may be only applied to paths having a non null curvature (Pettersen and Nijmeijer 1998*b*). Basically these limitations arise from the fact that a feedback-linearization approach is always attempted while these system models cannot be feedback-linearized. The present time-invariant steering solution takes explicitly into account the lateral distance of the vehicle from the desired

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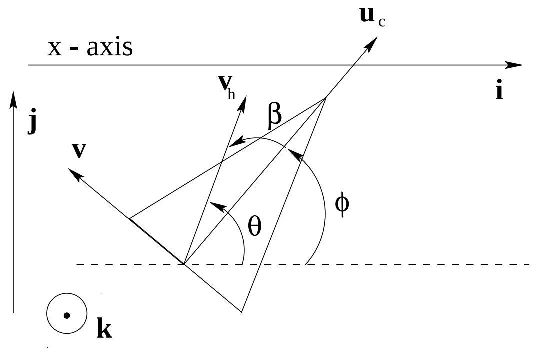

Fig. 1. The model

linear path and it guarantees that it is globally asymptotically stabilized to zero together with the sway velocity and heading direction (in the absence of constant sway disturbances) in spite of having non-actuated sway. To the best of the authors knowledge no other smooth time-invariant solution to this problem was suggested in the literature yet while a time-varying one that guarantees global practical convergence is reported in (Pettersen and Nijmeijer 1998*a*).

The controller is based on the complete nonlinear model where the sway axis is not neglected. The major difficulty in designing such kind of controller is due to the fact that feedback linearization cannot be applied to this problem (Pettersen and Nijmeijer 1998*b*) (Fantoni *et al.* 1999) (Indiveri December 1999). On the other hand the system is controllable and Brocketts Theorem (Brockett 1983) does not prevent the existence of a smooth time-invariant closed loop stabilizing solution. Thus a novel design approach recently shown to be effective in designing closed loop nonlinear position controllers for the $2D$ and $3D$ unicycle models (Aicardi *et al.* 2000*a*) (Aicardi *et al.* 2000*b*) has been applied. The control synthesis procedure is made of two major steps: first a smooth velocity vector field is defined such that an ideal point moving with such velocity would exponentially converge on the desired linear path. Then a steering law is defined such that the underactuated vehicle is exponentially parallel to the previously defined vector field.

In section (2) the model is described, in section (3) the steering law is derived and in section (4) its stability is proven. Section (5) describes a possible dynamic extension of the kinematic control solution, section (6) addresses robustness issues while concluding remarks are finally reported in section (7).

2. THE MARINE VEHICLE MODEL

With reference to figure (1) the model can be written as:

$$\begin{aligned}
\dot{v} &= -a\, u_c\, r - b\, v - c\, v\, |v| \\
\dot{y} &= v \cos\phi + u_c \sin\phi \\
\dot{\phi} &= r \\
a &= \frac{m_{11}}{m_{22}} \;\; ; \;\; b = \frac{d_v}{m_{22}} \;\; ; \;\; c = \frac{d_{v|v|}}{m_{22}}
\end{aligned} \tag{1}$$

being $u_c > 0$ the constant surge velocity, v the non-actuated sway velocity, m_{11} and m_{22} the inertia parameters (including the hydrodynamic added mass components) in the surge and sway directions, d_v and $d_{v|v|}$ the linear and quadratic sway drag coefficients, ϕ the vehicles heading, r the yaw angular velocity and y the lateral distance from the reference course given by the x axis of the fixed frame. The control objective is to design a smooth, closed-loop, time-invariant control law for r such that the state vector $\mathbf{x} = (v, y, \phi)^T \in \mathcal{R}^3$ is stabilized to zero. In section (5) it will be shown that having found such solution the dynamics of the yaw axis may be also easily included so that the control input is the yaw torque rather than the angular velocity. The controllability of such model, which is an obvious property to every seaman, can be rigorously proven together with the fact that it does not admit any input-state feedback linearization transformation as accounted in (Indiveri December 1999). In particular writing the model (1) in standard nonlinear control form, i.e. $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u,\ u = r, \mathbf{x} = (v, y, \phi)^T$, then the necessary condition for the existence of a feedback linearization transformation, namely that the set $\{\mathbf{g}, [\mathbf{f}, \mathbf{g}]\}$ is involutive (Khalil 1996)(Slotine and Li 1991), is not fulfilled (Indiveri December 1999). On the contrary as $\{\mathbf{g}, [\mathbf{f}, \mathbf{g}], [\mathbf{f}, [\mathbf{f}, \mathbf{g}]]\}$ span $\mathcal{R}^3$ the system is controllable (Indiveri December 1999).

It should be noticed that the apparently similar problem of stabilizing the same underactuated marine vehicle model in a point rather than on a linear course does not admit a smooth feedback solution because of Brocketts Theorem (Brockett 1983) and thus either time-varying (Pettersen and Egeland 1996) (Pettersen and Egeland 1997) or discontinuous (Fantoni et al. 1999) solutions must be considered. On the contrary the present model is not affected by Brocketts result as the mapping (1) $\gamma : (\mathbf{x}, r) \longmapsto \dot{\mathbf{x}}$ is onto $(0, 0, 0)^T$.

3. THE CONTROL LAW

Consider the smooth velocity vector field $\mathbf{v}_h$:

$$\mathbf{v}_h = u_c\, \mathbf{i} - ky\, \mathbf{j} \;\; : \;\; k > 0 \tag{2}$$

being $\mathbf{i}$ and $\mathbf{j}$ the unit vectors of the x and y fixed reference axis as shown in figure (1) and k a positive constant gain. An ideal point (x, y) moving with velocity $\mathbf{v}_h$ would exponentially converge on the x axis and proceed along it with asymptotic velocity $u_c\,\mathbf{i}$. Unfortunately in general the surge axis, i.e. the only actuated motion axis of the given system, will not be parallel to the $\mathbf{v}_h$ field: the basic idea underlying the proposed control strategy is to steer the marine vehicle such that its stern-bow axis (surge) is exponentially stabilized along the direction of $\mathbf{v}_h$. It will be shown that with a suitable choice of k this strategy indeed guarantees global asymptotic stability of state vector $\mathbf{x} = (v, y, \phi)^T$ to zero. Calling θ the angle between $\mathbf{i}$ and $\mathbf{v}_h$ and

$$\beta = \theta - \phi \tag{3}$$

the one between the axis of the vehicle and $\mathbf{v}_h$ the control objective for the design of $r = \dot{\phi}$ is to drive β exponentially to zero. This task is accomplished with a time constant $1/\gamma_\beta$ choosing

$$r = \gamma_\beta\,\beta + \dot{\theta} \quad : \quad \gamma_\beta > 0. \tag{4}$$

The time derivative of θ may be evaluated noticing that

$$\mathbf{i} \cdot \mathbf{v}_h = \|\mathbf{v}_h\| \cos\theta = \sqrt{u_c^2 + k^2 y^2}\cos\theta = u_c \tag{5}$$

$$\begin{aligned} \mathbf{i} \wedge \mathbf{v}_h &= \mathbf{k}\,\|\mathbf{v}_h\| \sin\theta = \\ &= \mathbf{k}\,\sqrt{u_c^2 + k^2 y^2}\sin\theta = -\mathbf{k}\,ky \end{aligned} \tag{6}$$

$$\Rightarrow \theta \in [-\pi/2, \pi/2]\ \forall\, t \tag{7}$$

being $\mathbf{k} = \mathbf{i} \wedge \mathbf{j}$ the unit vector of the z axis of the fixed reference. Taking the time derivative of equation (5) and using equations (6) and (1) it follows that

$$\dot{\theta} = -\mu\,(u_c \sin\phi + v\cos\phi) \tag{8}$$

being

$$\mu = \frac{k u_c}{u_c^2 + k^2 y^2} > 0. \tag{9}$$

Replacing equation (8) and (9) in equation (4) yields

$$r = \gamma_\beta\,\beta - \frac{k u_c (u_c \sin\phi + v\cos\phi)}{u_c^2 + k^2 y^2}. \tag{10}$$

This angular velocity is bounded and well defined in the whole state space and by construction it guarantees global exponential convergence of β to zero: under its effect the vehicle will be exponentially parallel to the field $\mathbf{v}_h$ no matter its initial condition.

4. CONVERGENCE ANALYSIS

By construction the angle β tends globally and exponentially to zero and as a consequence ϕ tends globally and exponentially to θ. Once that this has occurred the variables y and ϕ are coupled by equation (5) which shows that in the limit $\beta \to 0$ $\Rightarrow y = 0 \Leftrightarrow \theta = 0$ or equivalently $\phi = 0$. As a consequence global asymptotic (i.e. $\beta \to 0$ which occurs exponentially in time) stability of the state $\mathbf{x} = (v, y, \phi)^T$ is guaranteed if $(0, 0)^T$ is shown to be a globally stable equilibrium point for the vector $(v, \theta)^T$ in the limit $\beta \to 0$. Notice that as β tends to zero $\mathbf{x} = (v, y, \phi)^T$ will remain bounded as given the model (1) and the control (10) neither of the state variables admits a finite escape time. Replacing equation (10) in the first of equations (1), taking the limit $\beta \to 0$ it follows that:

$$\lim_{\beta \to 0} \begin{pmatrix} \dot{v} \\ \dot{\theta} \end{pmatrix} = \mathbf{A}(t) \begin{pmatrix} v \\ \theta \end{pmatrix} \tag{11}$$

being $\mathbf{A}(t)$ defined as

$$\mathbf{A}(t) = \begin{bmatrix} a u_c \mu \cos\theta - b - c|v| & a u_c^2 \mu \dfrac{\sin\theta}{\theta} \\ -\mu\cos\theta & -\mu u_c \dfrac{\sin\theta}{\theta} \end{bmatrix} \tag{12}$$

The linear time varying (LTV) system (11) is asymptotically stable if the eigenvalues of the matrix $1/2(\mathbf{A}(t) + \mathbf{A}^T(t))$ are shown to be smaller than a strictly negative constant at all times (Slotine and Li 1991). The characteristic polynomial of the matrix $1/2(\mathbf{A}(t) + \mathbf{A}^T(t))$ is given by $\lambda^2 + B\lambda + C = 0$ being

$$B = -a u_c \mu \cos\theta + b + c|v| + \mu u_c \frac{\sin\theta}{\theta} \tag{13}$$

$$\begin{aligned} C = &-\mu u_c \frac{\sin\theta}{\theta}(a u_c \mu \cos\theta - b - c|v|) \\ &- \frac{\mu^2}{4}\left(a u_c^2 \frac{\sin\theta}{\theta} - \cos\theta\right)^2. \end{aligned} \tag{14}$$

The eigenvalues $\lambda_\pm = 1/2(-B \pm \sqrt{B^2 - 4C})$ of $1/2(\mathbf{A}(t) + \mathbf{A}^T(t))$ are guaranteed to be smaller than a strictly negative constant at all times if

$$B \geq \varepsilon\ \cup\ C > 0.$$

From equations (7) and (9) it follows that

$$0 < \mu(t) \leq k/u_c \quad \forall \quad t \tag{15}$$

$$\frac{2}{\pi} \leq \frac{\sin\theta(t)}{\theta(t)} \leq 1 \quad \forall \quad \theta(t) \in [-\pi/2, \pi/2] \tag{16}$$

$$0 \leq \cos\theta(t) \leq 1 \quad \forall \quad \theta(t) \in [-\pi/2, \pi/2] \tag{17}$$

thus the condition $B \geq \varepsilon$ is satisfied if $b > ak$ being $\varepsilon = b - ak > 0$ and the condition $C > 0$ reduces to

$$u_c \frac{\sin\theta}{\theta}(-au_c\mu\cos\theta + b + c|v|) >$$
$$\frac{\mu}{4}\left(au_c^2\frac{\sin\theta}{\theta} - \cos\theta\right)^2. \tag{18}$$

Given equations (15), (16) and (17) a lower bound of the left hand side of the above inequality is given by

$$\frac{2u_c}{\pi}(b - ak)$$

and an upper bound of the right hand side is given by

$$\frac{k}{4u_c}\max\left\{(1 - au_c^2)^2, \frac{4a^2u_c^4}{\pi^2}\right\}$$

so that a sufficient condition for both $B \geq \varepsilon$ and $C > 0$ to hold is

$$b > k\left(a + \frac{\pi}{8u_c^2}\max\left\{(1 - au_c^2)^2, \frac{4a^2u_c^4}{\pi^2}\right\}\right). \tag{19}$$

This condition guarantees global asymptotic convergence to zero of the state $\mathbf{x} = (v, y, \phi)^T$ governed by the model (1) under the influence of the control law (10).

Notice moreover that condition (19) can be always met with a suitable choice of k for any non null value of the structural parameters a and b and of u_c.

The above proven global convergence and stability property of the proposed time-invariant controller represents the major result of this paper as to the best of the authors knowledge no other time-invariant feedback controller able to globally stabilize an underactuated marine system on a linear course has been presented in the literature so far.

5. YAW DYNAMICS

The smoothness of the control (10) enables the yaw dynamics to be explicitly taken into account: indeed if the yaw torque is assumed to be the control input rather than the yaw velocity r the following additional state equation needs to be added to the model (1):

$$m_{33}\,\dot{r} = (m_{11} - m_{22})u_c v - d_{33}\,r + \tau_3 \tag{20}$$

being m_{33}, d_{33} and τ_3 the yaw inertia moment (including added mass effects), the linear yaw drag coefficient and the input torque. Within this frame work r is now a new state variable. In particular τ_3 may be chosen as

$$\tau_3 = -(m_{11} - m_{22})u_c v + d_{33}\,r + \tau_N \tag{21}$$

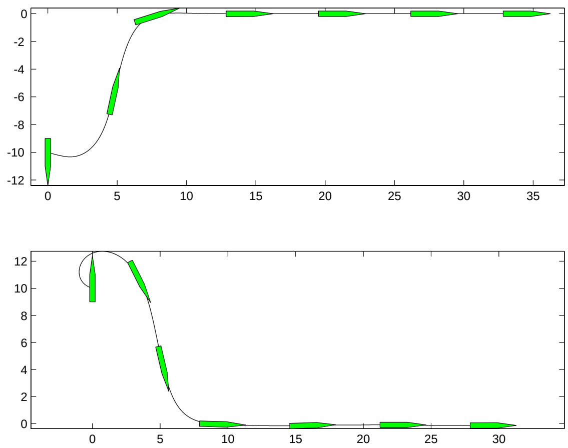

Fig. 2. Resulting paths and vehicle orientation for the kinematic (top) and dynamic (bottom) steering laws. The initial sway velocity is $v_0 = +3$ in both cases. The bow of the system corresponds to the point of the pencil-like drawn vehicle.

such that the yaw dynamics is reduced to

$$\dot{r} = \tau_N / m_{33}. \tag{22}$$

Calling now $\bar{r}$ the angular velocity given by equation (4), namely

$$\bar{r} = \gamma_\beta \beta + \dot{\theta}, \tag{23}$$

τ_N may be chosen such that the yaw rate r exponentially converges to the desired value $\bar{r}$. Indeed the Lyapunov function $V_N = (r - \bar{r})^2/2$ has a time derivative equal to:

$$\dot{V}_N = (r - \bar{r})(\tau_N/m_{33} - \dot{\bar{r}}) =$$
$$= -\gamma_\tau (r - \bar{r})^2 : \gamma_\tau > 0$$

if τ_N is chosen as

$$\tau_N = m_{33}\,\dot{\bar{r}} - \gamma_\tau(r - \bar{r}) \tag{24}$$
$$\dot{\bar{r}} = \gamma_\beta(\dot{\theta} - r) + \ddot{\theta} \tag{25}$$

where $\dot{\theta}$ is given by equation (8) and $\ddot{\theta}$ is its time derivative (bounded and well defined on the whole state space). Notice that given the model (1) both the state variables and the control signal will remain bounded as $r \to \bar{r}$. Replacing equations (23) and (25) in (24) the torque input τ_N

$$\tau_N = (m_{33}\gamma_\beta + \gamma_\tau)(\dot{\theta} - r) + \gamma_\beta\gamma_\tau(\theta - \phi) + m_{33}\ddot{\theta} \tag{26}$$

is found to have a PD (proportional - derivative) structure being the reference signal θ itself a function of the state. Straightforward calculations based on equations (1) and (8) show that $\ddot{\theta}$ may be written as:

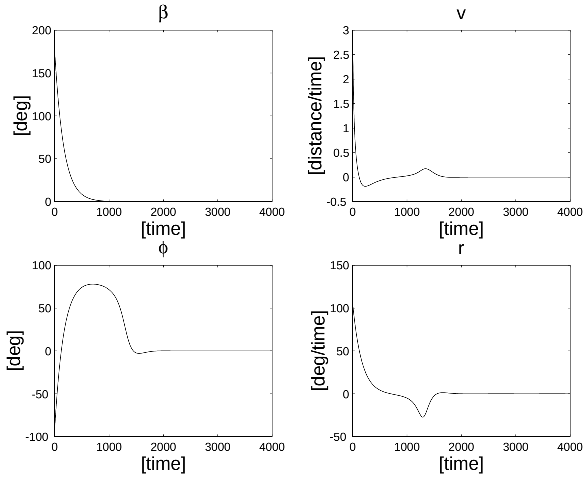

Fig. 3. Time history of β, v, ϕ and of the control r given by equation (10) relative to the path plotted in the top of figure (2).

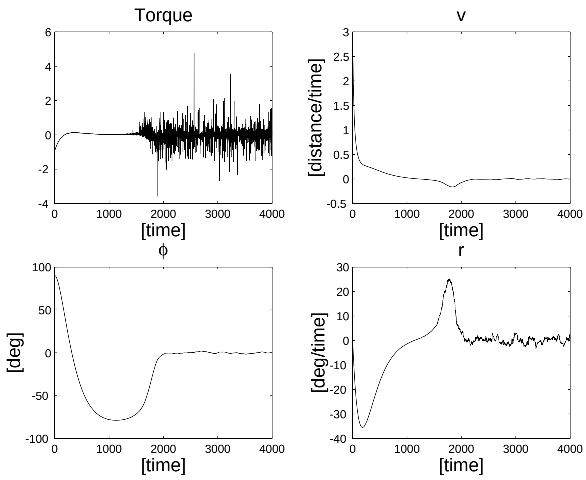

Fig. 4. Time history of the applied torque τ_N as given by equation (26) and of the state components v, ϕ and r relative to the path plotted in the bottom of figure (2). The effect of the simulated measurement noise is clearly visible.

$$\ddot{\theta} = 2y\frac{k}{u_c}\dot{\theta}^2 - \mu r[(1-a)u_c\cos\phi - v\sin\phi] \quad (27)$$
$$+ \mu(b + c|v|)v\cos\phi$$

being μ given by equation (9).

6. ROBUSTNESS ANALYSIS

Robustness with respect to measurement noise or parameter uncertainty can be analysed by standard means relying on the conventional Lyapunov theory and will not be developed here. More interesting is perhaps the robustness to environmental disturbances, in particular to wind or current forces acting on the non-actuated sway axis. Such disturbance may be modelled by a non null force of norm f_d pointing in direction ψ with respect to the fixed frame and the resulting sway dynamics is

$$\dot{v} = -a\,u_c\,r - b\,v - c\,v\,|v| + f_d\sin(\psi - \phi). \quad (28)$$

The equilibrium sway velocity v_{eq} would be given by the solution of the implicit equation:

$$b\,v_{eq} + c\,v_{eq}|v_{eq}| = f_d\sin(\psi - \phi_{eq}) \quad (29)$$

being ϕ_{eq} the equilibrium heading given by

$$\dot{y}|_{v_{eq},\phi_{eq}} = v_{eq}\cos\phi_{eq} + u_c\sin\phi_{eq} = 0 \Rightarrow \quad (30)$$
$$\phi_{eq} = \arctan\left(\frac{-v_{eq}}{u_c}\right). \quad (31)$$

Assuming ψ to be known f_d, which is generally unknown, may be estimated thanks to the measurements of v that have been assumed available throughout the paper. The linear time varying filter

$$\dot{\hat{v}} = -a\,u_c\,r - b\,v - c\,v|v| + \hat{f}_d\sin(\psi - \phi) + k_{obs}\,\tilde{v}$$
$$\dot{\hat{f}}_d = \sin(\psi - \phi)\,\tilde{v}$$
$$\tilde{v} = v - \hat{v} \;\; ; \;\; k_{obs} > 0$$

may be shown to be asymptotically convergent applying Barbalat's Lemma to $V = \tilde{v}^2 + \tilde{f}_d^2$, $\tilde{f}_d = f_d - \hat{f}_d$. Thus also ϕ_{eq} and v_{eq} may be estimated by equations (29) and (31) replacing $\hat{f}_d$ to f_d in equation (29). A steering law to converge on $y = 0$ in the presence of an unknown and non null f_d term in equation (28) may be designed with the same procedure outlined in the case $f_d = 0$ by replacing

$$\mathbf{v}_h = z\,\mathbf{i} - \mathbf{j}\,(ky + \lambda) \quad (32)$$
$$\lambda = \hat{v}_{eq}\cos\hat{\phi}_{eq} \quad (33)$$

$$z = \begin{cases} \left|\hat{v}_{eq}\,\dfrac{\cos\hat{\phi}_{eq}}{\tan\hat{\phi}_{eq}}\right| & \text{if } \hat{\phi}_{eq} \neq 0 \\ u_c & \text{if } \hat{\phi}_{eq} = \hat{v}_{eq} = 0 \end{cases} \quad (34)$$

to the $\mathbf{v}_h$ field given by equation (2). Equation (32) has been actually derived imposing the angle between $\mathbf{v}_h|_{y=0}$ and $\mathbf{i}$ to be ϕ_{eq}. It should be noticed that equation (32) reduces to (2) when $v_{eq} = \phi_{eq} = 0$, i.e. when $f_d = 0$. The resulting steering law is equivalent to equations (4) and (8) where

$$\mu = \frac{kz}{z^2 + (ky + \lambda)^2} > 0$$

rather than equation (9). The complete convergence and stability analysis of this observer-based solution recalls the one reported in the case $f_d = 0$ and will be the subject of future work.

7. CONCLUSIONS

The above described steering laws (10) and (26) with $f_d = 0$ have been tested by simulations and some examples are here reported. Paths resulting from the application of these laws are visible in figure (2). The starting configurations for these two examples are $(x_0, y_0, \phi_0) = (0, -10, -\pi/2)$ for the kinematic case and $(x_0, y_0, \phi_0) = (0, 10, \pi/2)$ for the dynamic one. The parameters are fixed to $(a, b, c, k, \gamma_\beta) = (1/2, 1, 1, 1, 0.6)$, the surge velocity is $u_c = 1$ and the initial sway is $v_0 = +3$ in both cases. In the dynamic case the initial yaw speed and the gain γ_τ are $r_0 = 0$ and $\gamma_\tau = 1/2$. The simulated experiment relative to the kinematic steering law (10) refers to the ideal case in which the state is perfectly known. On the contrary in simulating the steering law (26) it has been assumed that the available measured values of the state $(v, y, \phi, r)^T$ were affected by zero mean normally distributed noise, i.e. in evaluating equation (26) the state $(v, y, \phi, r)^T$ has been replaced by $(\hat{v}, \hat{y}, \hat{\phi}, \hat{r})^T$ such that $(\hat{v}, \hat{y}, \hat{\phi}, \hat{r})^T = (v, y, \phi, r)^T + (\varepsilon_v, \varepsilon_y, \varepsilon_\phi, \varepsilon_r)^T$ being $\varepsilon_* \sim \mathcal{N}(0, \sigma_*)$. The standard deviations have been fixed to $\sigma_v = 1/2$ (remember that $u_c = 1$), $\sigma_y = 1/2$, $\sigma_\phi = 5$ $[deg]$ and $\sigma_r = 5$ $[deg/time]$. The time history of the variables β, v, ϕ and r for the kinematic case are reported in figure (3) while the control τ_N and the variables v, ϕ and r relative to the dynamic case are plotted versus time in figure (4).

A nonlinear, closed-loop, time-invariant, smooth steering law that guarantees global asymptotic convergence and stability of an underactuated marine vehicle on a linear course has been presented. The marine vehicle is assumed to have unactuated sway and to travel at constant surge speed. For such kind of model a steering law for both the kinematic case (i.e. the yaw speed is the control input) and the dynamic one (i.e. the yaw torque is the control input) have been discussed. Simulations are reported to show the qualitative behaviour of the solution and its effectiveness in the presence of considerable state measurement noise. While traditional marine system autopilots stabilize the heading angle only, the proposed solution takes explicitly into account the lateral distance from the desired linear path and in spite of having unactuated sway it guarantees that this distance together with the heading direction and sway velocity are globally asymptotically stabilized. Moreover traditional autopilots are designed on the basis of the linearized model, thus yielding only local results.

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