Proceedings of Acoustics '08 Paris
Palais des Congrès, Paris, France, 29 June-4 July 2008

Acoustical Methods for Azimuth, Range and Heading Estimation in Underwater Swarms

Navinda Kottege & Uwe R. Zimmer

Research School of Information Sciences and Engineering
Robotics Research Group
The Australian National University, ACT 0200, Australia
navinda@ieee.org | uwe.zimmer@ieee.org

Abstract *– Robotic swarms and specifically schools of semi-autonomous underwater vehicles are becoming more and more of a reality. An essential feature required in most such systems is an efficient and precise relative localisation system. Specific, common requirements include posture estimations of multiple neighbours, controllable sensing ranges, as well as high robustness and accuracy.*

The presented system is based on acoustically transmitted MLS (Maximum Length Sequence) -signals. Multiple receivers allow for precise azimuth and range measurements, while multiple transponders additionally allow for complete posture estimations of neighbouring vehicles. The short baselines (< 200 mm) given by the dimensions of the vehicles in the swarm make sub-sample interpolations in the post-cross-correlation phase necessary. Furthermore the deployment of low cost transducers and hydrophones in a broadband signal setup requires specific filtering methods.

For the experimental setup as depicted in detail in the article, a mean error $\mu_{\Delta\theta}$ in the bearing (azimuth) estimation θ is less than 0.3° throughout all experiments, while the mean error $\mu_{\Delta\alpha}$ in the heading estimation α of a neighbouring vehicle is always less than 5.0°. Experiments have been performed in many configurations in a inter-vehicle distance of up to 5 m, while the mean range error $\mu_{\Delta r}$ remains well below 10 mm. All experiments have been performed in sweet water.

Keywords: **Robotic Swarm; Relative localisation; Posture estimation; Maximum Length Sequence (MLS); Acoustic (sonar) measurement; Bearing; Heading; Range; Autonomous underwater vehicles (AUVs); Unmanned underwater vehicles (UUVs);**

1. Introduction

Enhanced, embodied autonomy in small submersibles enables the design and deployment of practical swarms of autonomous underwater vehicles (AUVs). The swarming paradigm requires for each vehicle location awareness of at least its near neighbours. The Serafina AUV swarming project [9, 11] additionally requires a localisation system which could cope with the dynamic and fast changing vehicle configurations while being small, reliable, robust, and energy efficient and not dependent on previously deployed acoustic beacons.

Figure 1: Prototypes of Serafina MkII AUVs

The short-range acoustical relative localisation system proposed here, uses hyperbolic and spherical localization concepts and provides each vehicles with the azimuth, range and heading of its near neighbours. The implementation utilises an acoustically transmitted MLS-signal which provides extremely high robustness against interference by stochastic and systematic disturbances which are typical for underwater environments. The azimuth is obtained via hyperbolic positioning with

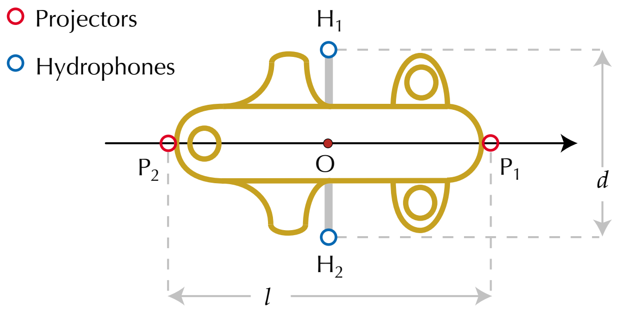

Figure 2: Transducer configuration on a Serafina hull.

improved resolution and accuracy with respect to conventional methods. Range estimation uses the implicit synchronisation provided by the underlying inter-vehicle communication scheduling system [9, 10] to measure the difference of time of arrivals (TOAs) of an acoustic and a long-wave radio signal. The heading estimation uses the intermediate sub-azimuths and sub-ranges produced by the azimuth and range estimation schemes mentioned above.

2. Technical description of the system

As it was introduced in [6] and [7] an MLS signal is transmitted acoustically from the bow and stern projectors of the sender AUV in sequence and the observer AUVs receive this signals on pairs of hydrophones. Figure 2 shows the configuration of projectors and hydrophones on a Serafina AUV hull.

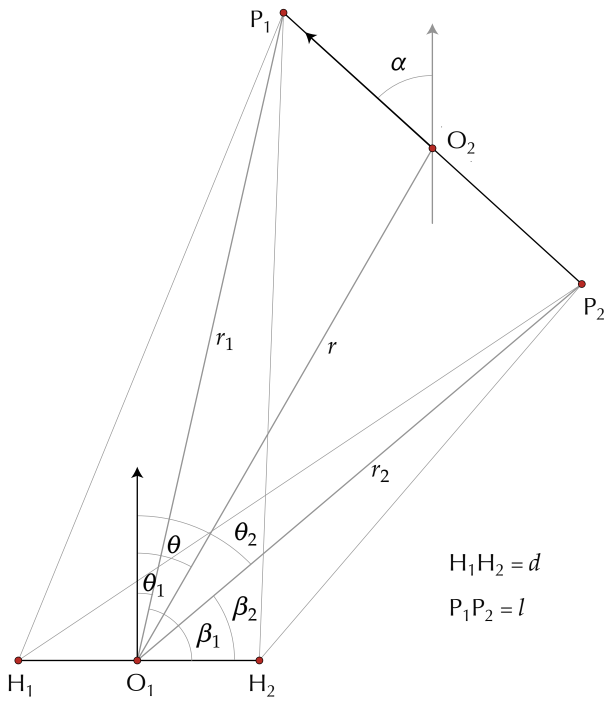

Figure 3: Geometric description of the projector and hydrophone configuration

The sending of the MLS signals are synchronised with the scheduling scheme used by the underlying longwave radio communication system explained in [10] and [9] which guarantees that there is only one sending event in a given local neighbourhood. Each of these 'sending events' corresponding to a time-step in the sending schedule consists of two MLS chirps, one emitted from the bow projector, the other from the stern projector. This results in two pairs of signal channels being received by each observer vehicle per sending event.

2-1. Azimuth estimation

Each of the two received signal channel pairs are cross-correlated and the position of the peak in the cross-correlogram is found using a local maxima search centred around the position of the peak in the previous estimation step. This process is bootstrapped by initially using the position of the absolute maximum peak position. By limiting the local maxima search neighbourhood (peak tracking), it was possible to avoid outliers caused by interference and reflected signals as explained in detail in [8]. The peak position found in this manner is further refined using a cubic spline interpolation scheme which provides sub-sample resolution for the position estimate.

If the sample-domain relative delays between the two cross-correlated channels corresponding to the two chirps are τ_0^1 and τ_0^2, they are related to the acoustic path length differences δ_1 and δ_2 (between the two hydrophones) as:

$$\delta_i = \frac{\tau_o^i v}{f_S}, i = 1,2 \tag{1}$$

where v is the speed of sound in water and f_S is the sampling frequency of the analogue to digital converter.

The angles of arrival for the two signals originating at the two ends of the sender AUV (Figure 3) is calculated as follows:

$$\beta_i = \tan^{-1}\left(\frac{\sqrt{d^2 - \delta_i^2}}{\delta_i}\right), i = 1,2 \tag{2}$$

The corresponding sub-azimuths θ_1 and θ_2 are obtained using the following:

$$\theta_i = [\beta_i - 90^\circ]_{adj}, i = 1,2 \tag{3}$$

with the adjustment function defined as:

$$[x]_{adj} = \begin{cases} x, -180^\circ < x \leq 180^\circ \\ sgn(x)(|x| - 360^\circ), \text{otherwise} \end{cases} \tag{4}$$

The main azimuth θ is calculated as the average between the two sub-azimuths as:

$$\theta = \frac{\theta_1 + \theta_2}{2} \tag{5}$$

The relationship between the angular quantities mentioned above are depicted in Figure 3.

2-2. Range estimation

Since the acoustic signals are synchronised with the communication scheduling scheme, the time elapsed between the start of a time step and the actual arrival of the acoustic signal constitutes the time of flight (TOF) of the MLS chirp between the sender's projector and the observer's hydrophone. A modified matched filtering technique [8] is used to detect the four TOFs corresponding to each of the two hydrophones receiving the two chirps emitted by the two projectors. Once again, the sample domain peak positions obtained via cross-correlating each of the received channels with a pre-recorded replica signal is refined using sub-sample interpolation scheme. These are then converted to a time-domain TOF using

$$t_{ij} = \frac{\tau_0^{ij}}{f_s}, i,j = 1,2 \tag{6}$$

and to a distance using

$$r_{ij} = t_{ij} v, i,j = 1,2 \tag{7}$$

where v is the speed of sound in water and f_s is the sampling frequency of the analogue to digital converter. According to Figure 3, r_{11}, r_{12}, r_{21} and r_{22} are denoted by the distances P_1H_1, P_1H_2, P_2H_1 and P_2H_2 respectively. The sub-ranges r_1 (P_1O_1) and r_2 (P_2O_1) are then calculated as:

$$r_i = \sqrt{\frac{r_{i1}^2 + r_{i2}^2}{2} - \left(\frac{d}{2}\right)^2}, i = 1,2 \tag{8}$$

and the main range is given by:

$$r = \sqrt{\frac{r_1^2 + r_2^2}{2} - \left(\frac{l}{2}\right)^2} \tag{9}$$

In these instances, d denotes the base distance between the hydrophones while l denotes the spacing between the projectors.

2-3. Heading estimation

The two sub-azimuths θ_1 and θ_2 and the two sub-ranges r_1 and r_2 obtained by the above azimuth and range estimation schemes are used to calculate the heading (orientation) of the sender as follows:

$$\alpha = \left[\tan^{-1}\left(\frac{r_1 \sin\theta_1 - r_2 \sin\theta_2}{r_1 \cos\theta_1 - r_2 \cos\theta_2}\right)\right]_{adj} \tag{10}$$

where the adjustment function is the same as defined in (4). As depicted in Figure 3, the heading

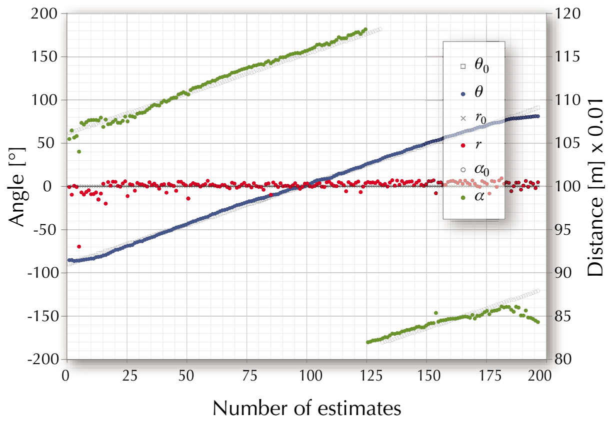

Figure 4: Estimates for azimuth θ, range r and heading α

angle α is the relative rotation between the coordinate frames fixed on the sender and the observer AUVs.

2-4. Error bounds

The theoretical error bounds for each of the quantities, azimuth θ, range r and heading α can be obtained by applying the general error propagation formula to (5), (9) and (10). The resulting error bounds are as follows:

$$\Delta\theta = \pm\frac{v\Delta\tau}{2 f_s d}\sqrt{\frac{1}{\cos^2\theta_1} + \frac{1}{\cos^2\theta_2}} \tag{11}$$

$$\Delta r = \pm\frac{v\Delta\tau}{f_s r}\sqrt{4r^2 + l^2 + d^2} \tag{12}$$

$$\Delta\alpha = \pm\frac{v\Delta\tau}{f_s l^2}\sqrt{\frac{\sin^2(\theta_1 - \theta_2)}{2}X + \frac{1}{d^2}Y} \tag{13}$$

where

$$\begin{aligned} X &= \left[4\left(r_1^2 + r_2^2\right) + \frac{d^2}{r_1^2 r_2^2}\left(r_1^4 + r_2^4\right)\right] \\ Y &= \left[\frac{(l^2 - r_2^2)^2}{\cos^2\theta_1} + \frac{(l^2 - r_1^2)^2}{\cos^2\theta_2}\right] \end{aligned} \tag{14}$$

As explained in [8], the azimuth error reaches a minimum in the vicinity of $\theta = 0^\circ$ while increasing rapidly close to $\pm 90^\circ$. The range error remains constant for distances greater than l while the heading error deteriorates with increasing range.

3. Experiments

In order to evaluate the performance of the estimates produced using the formulations presented earlier, a number of experiments were carried out. Transducers were mounted on Serafina mock-up hulls to represent the sender and observer AUVs and were moved relative to each other in the ANU

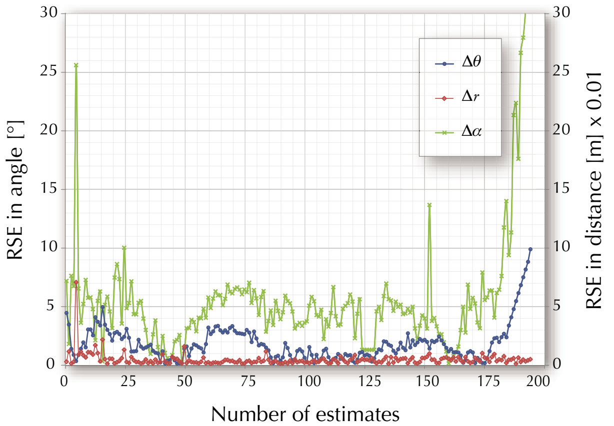

Figure 5: Unbiased root squared errors (RSE) in estimates for azimuth θ, range r and heading α

test tank[1]. A robotic gantry was placed on top of the tank and the sender and observer rigs (hulls mounted with transducers attached to a shaft) were connected to it. The motion of the gantry was pre-programmed and its angular and linear positions were used as the *'ground truth'* values with which the estimated quantities were compared.

During the experiments the base distance between hydrophones d was 0.3 m, the projector spacing l was 0.5 m and the sampling frequency f_s used was 96 kHz. The speed of sound in water which was calculated using the formula given in [3] was 1497 $\mathrm{m\,s^{-1}}$ An update rate of 5.0 Hz was used throughout the experiments.

1 *Cylindrical tank with corrugated metal walls filled with tap water. Diameter 4.2 m, depth 1.5 m.*

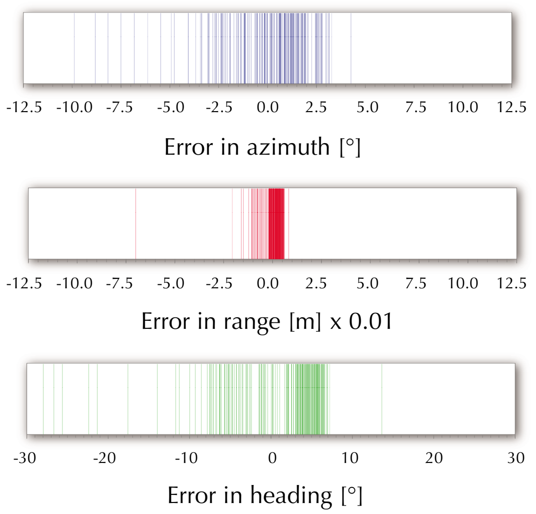

Figure 6: Estimation errors with bias for azimuth θ,range r and heading α

3-1. Results

Out of many experiments conducted with different configurations, Figure 4 plots the estimates for azimuth θ, range r and heading α produced by the localisation system for an explicit azimuth variation of θ: $-90^\circ \rightarrow 0^\circ \rightarrow 90^\circ$ along with the corresponding ground truth values θ_0, r_0 and α_0. This was achieved by a rotation of the observer rig while keeping the sender rig stationary at an angle corresponding to a heading of $\alpha = 150^\circ$ when the azimuth was at 0°. The range was kept constant at $r = 1.00\,\mathrm{m}$.

3-2. Errors and analysis

The deviation of the estimated quantities from the respective *'ground truth'* values are present themselves as estimation errors. In the following formulae ϕ is used as a placeholder for θ, r and α and the deviation of estimates with bias is given by:

$$\Delta\phi_i^{\text{biased}} = \phi_i - \phi_{0_i} \tag{15}$$

where ϕ_i is the estimate and ϕ_{0_i} the corresponding *'ground truth'* value at estimation step i. The mean $\mu_{\Delta\phi}$ and standard deviation $\sigma_{\Delta\phi}$ of the estimation error for m estimation steps are:

$$\mu_{\Delta\phi} = \frac{1}{m}\sum_{i=1}^{m}(\phi_i - \phi_{0_i}) \tag{16}$$

$$\sigma_{\Delta\phi} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\Delta\phi_i^{\text{biased}} - \mu_{\Delta\phi})^2} \tag{17}$$

The unbiased root squared error (RSE) of the estimate at estimation step i is

$$\Delta\phi_i = \sqrt{(\Delta\phi_i^{\text{biased}} - \mu_{\Delta\phi})^2} \tag{18}$$

The mean of the RSE which is equivalent to the average deviation of estimates is given by:

$$\overline{\Delta\phi} = \frac{1}{m}\sum_{i=1}^{m}\Delta\phi_i \tag{19}$$

Figure 5 plots the RSEs of estimates depicted in Figure 4 while Figure 6 plots the errors with bias. The statistical behaviour of these estimation errors are presented in Table 1.

4. Conclusions

In the azimuth variation experiment presented earlier, the errors associated with the azimuth estimate affects the heading estimate. Due to this, the

Azimuth error	Range error	Heading error
$\sigma_{\Delta\phi} = 2.39^\circ$	$\sigma_{\Delta r} = 6.77 \times 10^{-3}\mathrm{m}$	$\sigma_{\Delta\alpha} = 7.59^\circ$
$\mu_{\Delta\theta} = 0.00^\circ$	$\mu_{\Delta r} = 0.64 \times 10^{-3}\mathrm{m}$	$\mu_{\Delta\alpha} = 0.62^\circ$
$\overline{\Delta\theta} = 1.83^\circ$	$\overline{\Delta r} = 3.65 \times 10^{-3}\mathrm{m}$	$\overline{\Delta\alpha} = 5.40^\circ$

Table 1: Std. deviations, means and average deviations

errors in the azimuth estimate in the vicinity of 90° contribute to a proportional deviation in the errors for its dependant heading measurement.

The accuracy and precision of the estimates can be inferred by observing the mean errors and average deviations presented earlier. For all experiments conducted in the test tank (including the one presented) the mean error for azimuth remained less than 0.3° and the average deviation was under 2.0°. Estimation errors near the limits of $\pm 90^\circ$ remain well below 15.0° despite the theoretical formulae given in (11) suggesting an infinite error. For range estimates, the absolute mean error remained well below 1.0×10^{-2}m while the average deviation was at most 5.25×10^{-2}m during the experiments. Heading estimates displayed an absolute mean error of less than 5.0° in all test tank experiments while the maximum average deviation was 5.4°.

The localisation scheme presented in this paper demonstrates a higher degree of accuracy and precision of estimates when compared to other available systems addressing the problem of localisation for small AUVs including those mentioned and described [1], [2] and [5]. This is especially significant when considering the low power requirements, update rates, cost and scale of the system as well as the swarming paradigm which motivated the development.

References

[1] Baccou, P., Jouvencel, B., Creuze, V. and Rabaud, C.; Cooperative positioning and navigation for multiple AUV operations, In proceedings of MTS/IEEE Oceans 2001; Honolulu, HI, USA, November 2001, pp. 1816-1821.

[2] Baker, B. N., Odell, D. L., Anderson, M. J., Bean, T. A. and Edwards, D. B.; A new procedure for simultaneous navigations of multiple AUVs, In proceedings of MTS/IEEE Oceans 2005; Washington, DC, USA, September 2005, pp. 1-4.

[3] Coppens, A.B.; Simple equations for the speed of sound in Neptunian waters, *Journal of the Acoustical Society of America*; 69(3), March 1981, pp. 862-863.

[4] Deffenbaugh, M., Bellingham, J.G. & Schmidt, H.; *The relationship between spherical and hyperbolic positioning*, in proceedings of MTS/IEEE Oceans 1996; Fort Lauderdale, FL, USA, September 1996.

[5] Freitag, L., Johnson, M., Grund, M., Singh, S. and Preisig, J.; Integrated acoustic communication and navigation for multiple UUVs, In proceedings of MTS/IEEE Oceans 2001; Honolulu, HI, USA, November 2001, pp. 2065-2070.

[6] Kottege, N. & Zimmer, U. R.; ***Relative localisation for AUV swarms***, in proceedings of the IEEE International Symposium on Underwater Technology 2007 (SUT '07); Tokyo, Japan, April 2007.

[7] Kottege, N. & Zimmer, U. R.; *MLS Based Distributed, Bearing, Range and Posture Estimation for Schools of Submersibles*, Experimental Robotics; Springer: Berlin; Heidelberg, March 2008, pp. 377-385.

[8] Kottege, N.; *Underwater Acoustic Localisation in the context of Autonomous Submersibles*; submitted at The Australian National University, Canberra, ACT, Australia, April 2008.

[9] Schill, F.; *Distributed Communication in Swarms of Autonomous Underwater Vehicles*; PhD thesis, The Australian National University, Canberra, ACT, Australia, July 2007.

[10] Schill, F. & Zimmer, U. R.; *Pruning local schedules for efficient swarm communication*, in proceedings of the IEEE International Symposium on Underwater Technology 2007 (SUT '07); Tokyo, Japan, April 2007.

[11] http://serafina.anu.edu.au